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CHAPTER 6

SOFT-SWITCHING DC-DC CONVERTERS

A new class of dc-to-dc converters known in the literature as soft-switching resonant converters have been thoroughly investigated in recent years. Generally speaking, soft-switching means the one or more power switches in a dc-dc converter have either turn-ON or turn-OFF switching losses eliminated. This is on contrast to hard-switching when both turn-ON and turn-OFF of the power switches are employed at high current and high voltage levels. One approach is to create a full resonance phenomenon within the converter through series or parallel combinations of resonant components. Resultant converters are known generally as resonant converters. Another approach is to use the conventional PWM converters, buck, boost, buck-boost, Cuk, SPEC, and then replace the switch by a resonant switch that accomplish the same loss elimination mentioned above. Because of the nature of the PWM circuit, resonance occurs for a shorter time interval than the full resonance case. As a result, this class of converters that combines resonance and PWM is appropriately known as quasi-resonant converters. In this chapter, our focus will be on the later method, mainly using the resonance PWM switch to achieve soft-switching. For simplicity, here we used the term soft-switching to refer to dc-dc converters quasi resonant converters and other topologies that employ resonance to reduce switching losses. Two major techniques are generally employed to achieve soft-switching: Zero-Current-Switching (ZCS) and Zero-Voltage-Switching (ZVS). This chapter will focus on well-known types of PWM resonant switches and their steady state analyses.

6.1 Types of DC-DC Converters

As we have shown in previous chapters, linear-mode and switch-mode type converters have been used widely in the design of commercial dc-to-dc power supplies. The linear power supplies offer the designer four major advantages: simplicity in design, no electrical noise in its output, fast dynamic response time, and low cost. Whereas, their applications are limited due to several disadvantages: (1) The input voltage is at least two or three volts higher than the output voltage because it can only be used as the step down regulator, (2) each regulator is limited to only one output, and (3) low efficiency when compared to other switching regulators (30% to 60% for an output voltage less than 20V).

On the other hand, high frequency pulse width modulation (PWM) switching regulator circumvents all the linear regulator’s shortcomings: 1) Higher efficiency (68% to 90%), 2) since power transistors operate in their most efficient point’s cutoff and saturation, it allows for power densities of around 40 to 50 W/in\(^3\), 3) multi-output applications, and 4) the size and the cost are much lower when compared to the linear power supplies, especially at high power levels.
However, PWM switching converters still have several limitations among them: 1) more circuit complexity when compared to the linear power supplies, 2) High Electromagnetic Interference (EMI), and 3) because of high stress levels on power semiconductors devices, their switching speeds are limited to under 100kHz.

Recently, a third generation of power converters was introduced in literature in the late eighties. This group of topologies is known as soft-switching resonant converters. Unlike linear and switched-mode converters, the potential advantage of soft-switching resonant converter are reduced power losses, therefore, achieving high switching frequency and high power density, while maintaining high efficiency. Moreover, due to the higher switching frequency, such converters exhibit faster transient responses. Today’s soft-switching techniques are used in the design of both high frequency dc-to-dc conversion and high frequency dc-to-ac conversion. Due to the scope of this text book only the first application is discussed here.

The Resonant Concept:

Like switch mode dc-to-dc converter, resonant converters are used to convert dc-to-dc through an additional conversion stage: the resonant stage in which dc signal is converted to high frequency ac signal. The potential advantage of resonant converter include the natural commutation of power switches, resulting in low switching power dissipation and reduced component stresses, which in turns results in increased power efficiency and increased switching frequency; higher operating frequencies results in reduced size and weight of equipment and results in faster responses; possible reduction in EMI problems.

Since the size and weight of the magnetic components (inductors and transformers) and capacitors in a converter are inversely proportional to the converter switching frequency, many power converters have been designed at progressively higher frequencies in order to reduce excessive size and weight and obtain fast converter transients.

In recent years, the market demand for wide applications that need variable speed drives, highly regulated power supplies, uninterruptible power supplies, and the desire to have smaller size and lighter weight power electronics systems has been increased.

There are many soft witching techniques available in the literature to improve the switching behavior of dc-to-dc resonant converters. At the time of writing these words, intensive research in soft switching is under way to further improve efficiency with increased switching frequency of power electronic circuits.

From a circuit standpoint, a dc-to-dc resonant converter can be described by three major circuit blocks as shown in Fig. 6.1: The dc-to-ac input inversion circuit, the resonant energy buffer tank circuit, and the ac-to-dc output rectifying circuit. Typically, the dc-to-ac inversion is achieved by using a various types of switching network topologies. The resonant tank which serves as an energy buffer between the input and output is normally synthesized by using lossless frequency selective network. The purpose of that network is
to regulate the energy flow from the source to the load. Finally, the ac-to-dc conversion is achieved by incorporating rectifier circuits at the output section of the converter.

*Fig. 6.1 Typical block diagram of soft switching dc-to-dc converter*

**Resonant verses Conventional PWM:**

For many years, high efficiency power processing circuits has been achieved by operating power semiconductor devices in the switching mode, whereby switching devices are operated in either the *ON* or *OFF* states as in the PWM method. In PWM converters, switching of semiconductor devices normally occurs at high current levels. Therefore, when switching at high frequencies these converters are associated with high power dissipation in their switching devices. Furthermore, the PWM converters suffer from EMI caused by high frequency harmonic components associated with their quasi-square switching current and/or voltage waveforms with today’s power semiconductor and circuit technology, PWM converters can operate up to 200 kHz. Unfortunately, even though the technological advancement of PWM switch mode converters has resulted in faster switching devices, their operating frequency is limited by the reasons mentioned above.

In both techniques, the switching losses in the semiconductor devices are avoided due to the fact that current through or voltage across the switching device at switching point is equal to or near zero. This reduction in the switching loss allows the designer to attain a higher operating frequency without sacrificing converter efficiency. By doing so, the resonant converters show promise of achieving what could not be achieved by the PWM converters, that is the design of small size and weight converters. Currently, resonant power converters operating in the range of a few megahertz are available. Another advantage of resonant converters over PWM converters is the decrease of harmonic content in the converter voltage and current waveforms. Therefore, when the resonant and PWM converters are operated at the same power level and frequency, it is expected that the resonant converter will have lower harmonic emission.

**6.2 Classification of soft-switching Resonant Converters:**

The literature is extremely rich with resonant power electronic circuits used in application such as dc-to-dc and dc-to-ac resonant converters. There exist no general classification of resonant converter topologies.

There are several types of dc-to-dc converter that employ additional resonant stage, that have been explored in the open literature. These include:

- *Quasi-resonant converter (single ended):*
  - Zero-Current Switching (ZCS)
  - Zero-Voltage Switching (ZVS)
- *Full-resonant converters (Conventional)*
  - Series Resonant Converter (SRC).
• Parallel Resonant Converter (PRC).
• Quasi-Square-Wave (QSW) Converters:
  • Zero-Current Switching (ZCS)
  • Zero-Voltage Switching (ZVS)
• Zero-Clamped Topologies.
  • Zero-Clamped-Voltage (CV).
  • Zero Clamped-Current (CC).
• Class-E resonant converter.
• DC-Link Resonant Inverters.
• Multi-resonant Converters.
  • Zero-Current Switching (ZCS)
  • Zero-Voltage Switching (ZVS)
• Zero Transition Topologies:
  • Zero-Voltage Transition (ZVT).
  • Zero-Current Transition (ZCT).

Many other variations of soft-switching topologies exist today that are beyond the scope of this text. Since the scope of this text targets undergraduates electrical engineering students, we only focus on the quasi resonant type PWM converters.

### 6.3 Advantages and disadvantages of ZCS and ZVS

The major advantage of ZCS and ZVS quasi-resonant converters is that the power switch is turned-ON and -OFF at Zero-Voltage and Zero-Current, respectively. In ZCS topologies, the rectifying diode has ZVS operation. Whereas, in ZVS topologies, the rectifying diode has ZCS.

The second advantage is that both ZVS and ZCS utilize transformer leakage inductance and diode junction capacitors and output parasitic capacitor of the power switch.

The major disadvantage of ZVS and ZCS techniques is that they require variable-frequency control to regulate the output. This is undesirable since it complicates the control circuit and generate unwanted EMI harmonics, especially under wide load variations.

In ZCS, the power switch turns-OFF at zero current but at turn-ON, the converter still suffers from the capacitor turn-ON loss caused by the output capacitor of the power switch.

**Switching Loci:**

In general, most regulator converter switches need to turn ON or turn OFF the full load current at high voltage, resulting in what is known hard-switching. Figures 6.2 (a) and (b) show typical switching loci for a hard-switching converter with and without snubber circuits, respectively.
As stated before, in a soft-switching converter topology, an LC resonant network is added to shape the switching devices’ voltage or current waveform into quasi-sine wave in as such a way that zero voltage or current condition is created. This technique eliminates the turn-\textit{ON} or turn-\textit{OFF} loss associated with the charging or discharging of energy stored in the MOSFET parasitic junction capacitances. Figures 6.3(a) and (b) show typical switching loci for ZVS at turn-\textit{ON} and the ZCS at turn-\textit{OFF} cases.

\textbf{Fig. 6.3} ZVS at turn-\textit{ON}, (b) ZCS at turn-\textit{OFF}

\textbf{Switching Losses:}

As frequency of operation increases, the switching losses also increase. There are two types of switching losses:

1) At turn-\textit{OFF}, the power transformer leakage inductance produces high $\text{d}i/\text{d}t$ that results in high voltage spike across it.

2) At turn-\textit{ON}, the switching loss is mainly caused by the dissipation of energy stored in the output parasitic capacitor of the power switch.

Figures 6.4(a) and (b) show typical switching waveforms at turn-\textit{OFF} and turn–\textit{ON}, respectively.

\textbf{Fig. 6.4 Typical switching current, voltage and power losses waveforms at (a)Turn-\textit{OFF}, (b) Turn-\textit{ON}}

\textbf{6.4 ZERO-CURRENT SWITCHING TOPOLOGIES}

\textbf{6.4.1 The Resonant Switch:}

In this section, we will present one class of PWM converters that was introduced in the open literature in the late eighties based on the concept of using the conventional PWM switching along with an LC tank circuit. Depending on the inductor-capacitor arrangements, there are two possible types of resonant switch arrangements. The switch is either L-type or M-type and can be implemented as a half-wave or full-wave which correspond to whether the switch current can be unidirectional or bi-directional, respectively. Figure 6.5 shows the L-type in both half and full-wave implementation. Similarly, the M-type switch is shown as Fig. 6.6*.

\textbf{Fig. 6.5 Resonant Switch:(a) L-Type Switch, (b)Half-wave implementation, (c)Full-wave implementation.}

* The notations L-type and M-type were first used by the original authors.
The three conventional converter topologies, buck, boost and buck-boost, will be analyzed here. In all of these topologies, the LC tank form the resonant tank that make ZCS to occur. The buck, boost, and buck-boost converters are shown in Fig. 6.7(a), (b), and (c), respectively.

The detailed steady-state analysis of these converters is well known and was presented in details in Chapter 4. They show that there exist two \textit{ccm} modes of operation (as far as energy transfer is concerned): One mode during which energy is transferred from the source to the storage inductance, and the second mode during which energy is transferred from the storage inductance to the load. A low-pass LC filter is used in all three topologies to filter the harmonics from the fundamental. Using the two types of switch arrangements, it is possible to convert the above topologies into what is known as quasi-resonant converters.

6.4.2 Steady State Analysis

To simplify the steady-state analysis of the steady state condition for the above converters, there are some assumptions need to be made:

1. The filtering components \( L_o, L_{in}, L_F \) and \( C_o \) are very large when compared to the resonant components \( L \) and \( C \).
2. Output filter \( L_o - C_o - R \) is treated as a constant current source, \( I_o \).
3. Output filter \( C_o - R \) is treated as constant voltage source, \( V_o \).
4. Ideal switching devices and diodes.
5. Ideal reactive circuit components.

6.4.2.1 The Buck-Resonant Converter

Replacing the switch in Fig. 6.7(a) by the resonant type switch of Fig. 6.5(a), we obtain a new quasi-resonant PWM buck converter as shown in Fig. 6.8(a). Its simplified equivalent circuit is shown in Fig. 6.8(b).

It can be shown that there are four modes of operation under steady state condition as discussed below.

\[ \text{Mode I } [0 \leq t < t_1] : \]
Mode I starts at t=0 when S is turned ON. Since the switch was OFF prior to t = 0, it is clear that the diode must have been ON for t < 0 to carry the output inductor current. Hence, we assume for t > 0, both S and D are ON. The output current is equal to the constant current source, $I_o$, as shown in Fig. 6.9(a). In this mode, the capacitor voltage, $v_c$, is zero and the input voltage is equal to the inductor voltage as given by,

$$V_{in} = L \frac{di_L}{dt} \tag{6.1}$$

Integrating Eq. (6.1) from 0 to t, the inductor current $i_L$ is given by,

$$i_L(t) = \frac{V_{in}}{L} t \tag{6.2}$$

Equation (6.2) assumes zero initial condition for $i_L$. The current and voltage waveforms are given in Fig. 6.10.

As long as the inductor current is less than $I_o$, the diode will continue conducting and the capacitor voltage remains at zero. At time $t_1$, the inductor current becomes equal to $I_o$, the diode stops conducting, and the circuit enters Mode II. Evaluating Eq. (6.2) at t=$t_1$, we have,

$$I_o = \frac{V_{in}}{L} t_1 \tag{6.3}$$

Hence, the time interval $\Delta t_1 = t_1$ is given by

$$\Delta t_1 = t_1 = \frac{LI_o}{V_{in}} \tag{6.4}$$

This is the inductor current charging state.

**Mode II [$t_1 \leq t < t_2$]**

Mode II starts at $t_1$ when the diode open circuited as shown in Fig. 6.9(b) resulting in a resonant stage between L and C. During the time between $t_1$ and $t_2$, the switch remains ON, but the diode is OFF. The initial capacitor voltage remains zero, but the initial inductor current become has changed to $I_o$.

$$V_{in}$$

The first order differential equations that represent this mode are given by Eq. (6.5),
Next we express the inductor current in a second order differential equation for \( t > t_1 \) as given in Eq. (6.6).

\[
\frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = \frac{I_o}{LC}
\]  

(6.6)

From Eq. (6.6), the general solution for \( i_L(t) \) is given by,

\[
i_L(t) = A_1 \sin \omega_o(t - t_1) + A_2 \cos \omega_o(t - t_1) + A_3
\]  

(6.7)

where the resonant angular frequency is defined by,

\[
\omega_o = \sqrt{\frac{1}{LC}}
\]

Next we need to evaluate the constants \( A_1, A_2 \) and \( A_3 \). Recall, at \( t = t_1, i_L(t_1) = I_o \). Using this value, Eq. (6.7) becomes,

\[
A_2 + A_3 = I_o
\]

The other relation in terms of the constants is obtained from the following equation,

\[
L \frac{di_L(t)}{dt} = V_{in} - v_c(t)
\]

Substitute for \( i_L \) from Eq. (6.7) and since the capacitor voltage is zero at \( t_1 \), we have

\[
A_1 = \frac{V_{in}}{L \omega_o}
\]

and by taking the derivative of \( i_L \) again where \( L \frac{di_L^2(t_1)}{dt^2} = 0 \), result in the value of \( A_2 \) to be zero. Therefore, \( A_3 = I_o \).

By solving the above equations, \( i_L \) and \( v_c \) are given by,

\[
i_L(t) = I_o + \frac{V_{in}}{Z_o} \sin \omega_o(t - t_1)
\]  

(6.8)
\[ v_c(t) = V_n [1 - \cos(\omega_o (t - t_i))] \]  

(6.9)

where \( Z_o = \frac{L}{\sqrt{C}} \) is known as characteristic impedance.

This mode will last until \( t = t_2 \), when the transistor turns OFF because the inductor current reaches zero at this point. The peak inductor current occurs at \( t = t_{\text{max}}' \), where \( \omega_o (t_{\text{max}}' - t_i) = \pi / 2 \). When \( v_c(t) = V_{in} \), the peak inductor current is \( I_o + V_{in} / Z_o \). Moreover, the peak capacitor voltage occurs at \( t = t_{\text{max}}'' \), where \( \omega_o (t_{\text{max}}'' - t_i) = \pi \). When \( i_L(t) = I_o \), the peak capacitor voltage is \( 2V_{in} \).

The time interval in this mode can be derived at \( t = t_2 \) by setting \( i_L(t_2) = 0 \),

\[
\begin{align*}
  i_c(t_2) &= I_o + \frac{V_{in}}{Z_o} \sin(\omega_o (t_2 - t_i)) \\
           &= 0
\end{align*}
\]

(6.10)

therefore,

\[
\Delta t_2 = t_2 - t_1 = \frac{1}{\omega_o} \sin^{-1} \left( -\frac{Z_o I_o}{V_{in}} \right)
\]

(6.11)

Mode III starts at \( t = t_2 \), when the switch is turned OFF.

Mode III \([t_2 \leq t < t_3]\):

Fig. 6.9(c) Equivalent circuit for Mode III.

At \( t_2 \), the inductor current becomes zero, and the capacitor linearly discharge from \( v_c(t_2) \) to zero during \( t_2 \) to \( t_3 \). The diode remains OFF since its voltage is negative as shown in Fig. 6.9(c).

Now the initial value of \( i_L(t_2) \) is zero and the initial value of the capacitor at \( v_c(t_2) \) is \( V_{c2} \). The inductor has no current goes through it when the switch is OFF, so the capacitor current equals to \( I_o \) as given by,

\[
i_c = C \frac{dv_c}{dt} = -I_o
\]

(6.12)
The capacitor voltage \( v_c(t) \) is obtained by integrating Eq. (6.12) from \( t_2 \) to \( t \) with \( V_c(t_2) \) as initial value, resulting in Eq. (6.13),

\[
v_c(t) = -\frac{I_o}{C}(t-t_2) + V_c(t_2)
\]  

(6.13)

The initial value \( V_c(t_2) \) is obtained from Eq. (6.9) in the previous mode, to yield

\[
V_c(t_2) = V_{in} [1 - \cos \omega (t_2 - t_1)]
\]  

(6.14)

Substitute Eq. (6.14) into Eq. (6.13), we obtain,

\[
v_c(t) = -\frac{I_o}{C}(t-t_2) + V_{in}[1 - \cos \omega (t_2 - t_1)]
\]  

(6.15)

At \( t = t_3 \), the capacitor voltage becomes zero, and the equation for this time interval is given by,

\[
\Delta t_3 = t_3 - t_2 = \frac{C}{I_o} V_{in} [1 - \cos \omega (t_2 - t_1)]
\]  

(6.16)

At this point, the diode turns ON and the circuit enters Mode IV. In this mode, the capacitor voltage and inductor current remain at zero until the switch is turned ON again to repeat Mode I.

**Mode IV \([t_3 \leq t < t_4]\):**

*Fig. 6.9(d) Equivalent circuit for Mode IV.*

At this mode the switch remains OFF, but the diode starts conducting at \( t = t_3 \). Mode IV will continue as long as the switch is OFF, and the output current starts free-wheeling stage through the diode. The inductor current and the capacitor voltage are zero when the switch closes.

\[
i_i(t) = 0
\]

\[
v_c(t) = 0
\]

Therefore, there will be no power transfer during this mode. By turning ON the switch at \( t = t_4 \), the cycle will repeat these four modes. The dead time \( \Delta t_4 \) is given by,

\[
\Delta t_4 = T_s - \Delta t_1 - \Delta t_2 - \Delta t_3
\]  

(6.17)
Figure 6.10 shows the steady state waveforms for \(v_c\) and \(i_L\) for the buck converter with L-type switch.

**Fig. 6.10 Characteristic Waveform of buck-L-Type**

**Voltage Gain:**

In this section, we will derive the expression for the voltage gain, \(M = \frac{V_o}{V_{in}}\), in terms of the circuit parameters.

The average output voltage, \(V_o\), can be obtained by evaluating the following integral,

\[
V_o = \frac{1}{T_s} \int_0^{T_s} v_c(t) dt
\]  

(6.18)

Substitute for \(v_c(t)\) from Eqs. (6.9) and (6.13) for intervals \((t_2-t_1)\) and \((t_3-t_2)\), respectively, into Eq. (6.18) to yield,

\[
V_o = \frac{1}{T_s} \left[ \int_{t_1}^{t_2} V_{in} (1 - \cos \omega_o (t - t_1)) dt + \int_{t_2}^{t_3} \left( \frac{-I_o}{C} (t - t_2) + V_c (t_2) \right) dt \right]
\]

The voltage gain ratio is given by,

\[
\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[ (t_2 - t_1) - \frac{\sin \omega_o (t_2 - t_1)}{\omega_o} - \frac{I_o}{V_{in} C} \left( \frac{(t_3 - t_2)^2}{2} + V_c (t_2) (t_3 - t_2) \right) \right]
\]  

(6.19)

Substitute for \((t_2-t_1)\), \((t_3-t_2)\) and \(V_{in}(t_2)\) from Eqs. (6.11), (6.16) and (6.14), respectively into Eq. (6.19), to yield a closed form expression for \(M\) in terms of the circuit parameters. However, for illustration purposes, we will show next that the same expression can be obtained from the conservation of power law. The conservation of energy per switching cycle states that since the converter is assumed ideal, then the average input and output powers should be equal.

Next we evaluate the average input and output powers then equate them since the converter is assumed ideal.

The total input energy over one switching cycle is given by,

\[
E_{in} = \int_0^{T_s} i_{in} V_{in} \ dt
\]  

(6.20)

Since \(i_{in}\) equals to \(i_L(t)\), Eq. (6.20) is rewritten as,
Substitute for \( i_l(t) \) from Eqs. (6.2) and (6.8) into the above integrals, respectively, resulting in Eq. (6.21) to become,

\[
E_{in} = V_{in} \left[ \frac{V_{in}}{2L} t_1^2 + I_o(t_2 - t_1) + \frac{V_{in}}{Z_o \omega_o} \left[ \cos \omega_o(t_2 - t_1) - 1 \right] \right] \quad (6.22)
\]

Substitute for \( \cos \omega_o(t_2 - t_1) = 1 - \frac{I_o(t_3 - t_2)}{CV_{in}} \) from Mode II, Eq. (6.22) becomes

\[
E_{in} = V_{in} \left[ \frac{t_1}{2} I_o + I_o(t_2 - t_1) + \frac{V_{in}}{Z_o \omega_o} \left[ \frac{I_o(t_3 - t_2)}{CV_{in}} \right] \right] \quad (6.23)
\]

with \( Z_o \omega_o = \frac{1}{C} \), Eq. (6.23) becomes,

\[
E_{in} = V_{in} I_o \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \quad (6.24)
\]

The output energy over one cycle is obtained by evaluating Eq. (6.25),

\[
E_o = \int_{0}^{T_s} I_o V_o \, dt = I_o V_o T_s \quad (6.25)
\]

From the conservation of energy theory, equating the input and output energy expressions from Eq. (6.24) and (6.25), we have

\[
I_o V_o T_s = V_{in} I_o \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \quad (6.26)
\]

From Eq. (6.26) the voltage gain is expressed by,

\[
\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[ \frac{t_1}{2} + (t_2 - t_1) + (t_3 - t_2) \right] \quad (6.27)
\]
Substitute for $t_1$, $(t_2-t_1)$ and $(t_3-t_2)$ from Eqs. (6.4), (6.11), (6.16), respectively, into Eq. (6.27), the voltage gain becomes

$$
\frac{V_o}{V_{in}} = \frac{1}{T_s} \left[ \frac{LI_o}{2V_{in}} + \frac{1}{\omega_o} \sin^{-1} - \frac{Z_o I_o}{V_{in}} + \frac{CV_{in}}{I_o} \left[1 - \cos \omega_o (t_2 - t_1)\right] \right]
$$

(6.28)

To simplify and generalize the gain equation, the following normalized parameters are defined:

$$
M = \frac{V_o}{V_{in}} \quad \text{normalized output voltage} \quad (6.29a)
$$

$$
Q = \frac{R_o}{Z_o} \quad \text{normalized load} \quad (6.29b)
$$

$$
I_o = \frac{V_o}{R_o} \quad \text{average output current} \quad (6.29c)
$$

$$
f_{ns} = \frac{f_s}{f_o} \quad \text{normalized switching frequency} \quad (6.29d)
$$

By substituting Eq. (6.29) into Eq. (6.28), the final voltage gain is simplified into

$$
M = \frac{f_{ns}}{2\pi} \left[ \frac{M}{2Q} + \alpha + \frac{Q}{M} \left(1 - \cos \alpha\right) \right]
$$

(6.30)

where,

$$
\alpha = \sin^{-1} - \frac{M}{Q}
$$

(6.31)

A plot of control characteristics curve of $M$ vs. $f_{ns}$ under various normalized loads is given in Fig. 6.11

**Fig. 6.11: Control characteristic Curve of $M$ vs. $f_{ns}$ for the ZCS buck converter**

**Example 6.1**

Consider the following specification for a ZCS buck converter of Fig. 6.8(a). Assume the parameters are: $V_{in} = 25V$, $V_o = 12V$, $I_o = 1A$, $f_s = 250kHz$

Design for the resonant tank parameters $L$ and $C$ and calculate the peak inductor current, peak capacitor voltage. Determine the time interval for each mode.
Solution:

The voltage gain is \( M = \frac{V_o}{V_{in}} = \frac{12}{25} = 0.48 \). Let us select \( f_{ns} = 0.4 \). Next we determine \( Q \) from either the control characteristic curve of Fig. 6.11 or from the gain equation of Eq. (6.30). This results in approximately \( Q \approx 1 \). Since \( R_o = \frac{V_o}{I_o} \), the characteristic impedance is given by,

\[
Z_o = \frac{R_o}{Q} = 12 \Omega \\
= \sqrt{\frac{1}{LC}} = 12 \Omega 
\]

The second equation in terms of \( L \) and \( C \) is obtained from \( f_o \). From the normalized switching frequency, \( f_o \) may be given by,

\[
f_o = \frac{f_s}{f_{ns}} = 0.4 \\
f_o = \frac{f_s}{0.4} = 625 kHz
\]

In terms of the angular frequency, \( \omega_o \), we have,

\[
\omega_o = 2\pi f_o = \sqrt{\frac{1}{LC}} 
\]

Solving Eqs. (6.32) and (6.33) for \( L \) and \( C \) to yield,

\[
L = \frac{Z_o}{\omega_o} = \frac{12 \Omega}{2\pi \times 625 \times 10^3 \text{ rad/sec}} = 3.06 \times 10^{-6} \approx 3 \mu H \\
C = \frac{1}{Z_o \omega_o} = \frac{1}{12 \times 2 \times \pi \times 625 \times 10^3} = 0.02 \mu F
\]

the peak inductor current, is given by,
\[ I_{\text{I, peak}} = I_o + \frac{V_{\text{in}}}{Z_o} \]
\[ \approx 3 \, A \]

and the peak capacitor voltage is:

\[ v_{c, \text{peak}} = 2 \, V_{\text{in}} \]
\[ = 50 \, V \]

the time intervals are calculated from the following expression:

\[ t_1 = \frac{I_o L}{V_{\text{in}}} = \frac{(1 \, A) \times (3 \times 10^{-6} \, H)}{25 \, V} = 0.122 \, \mu s \]

\[ t_2 = t_1 + \frac{1}{\omega_o} \sin^{-1} \left( \frac{-Z_o I_o}{V_{\text{in}}} \right) \]
\[ \approx 0.122 + \frac{1}{2 \pi f_o} \sin^{-1} \left( \frac{-12 \times 1}{25} \right) \]
\[ \approx 0.795 \, \mu s \]

\[ t_3 = t_2 + \frac{C V_{\text{in}} (1 - \cos \omega_o (t_2 - t_1))}{I_o} \]
\[ = 0.795 + \frac{\left( 0.02 \times 25 \times 10^{-6} \right)(1 - \cos \omega_o 0.67)}{1 \, A} \]
\[ = 1.79 \, \mu s \]

For \( t'_{\text{max}} \) we have,

\[ \omega_o (t'_1 - t_1) = \frac{\pi}{2} \]
\[ t'_1 = \frac{\pi/2}{\omega_o} + t_1 \]
\[ = 0.4 \, \mu s + 0.122 \, \mu s \]
\[ = 0.522 \, \mu s \]
\[ t_4 = 4 \, \mu s = T_s \]

**Exercise 6.1**
Consider the ZCS bulk converter with the following parameters:
\( V_{in}=40\, \text{V}, \quad V_o=28.7\, \text{V} \) @ \( I_o=0.6\, \text{A}, \quad f_s=100\, \text{kHz}, \quad L=15\, \mu\text{H}, \quad \text{and} \quad C=60\, \mu\text{F} \). Determine the peak inductor current and the time at which the capacitor reaches its maximum voltage of 40V.

Ans 3.1A, 3.2\( \mu \)s.

Exercise 6.2:

Figure 6.12(a) shows another type of quasi-resonant buck converter that uses another resonant switch arrangement (M-type). Its simplified circuit is shown in Fig. 6.12(b). Drive the voltage gain equation, \( M = \frac{V_o}{V_{in}} \), for the steady state waveform shown in Fig. 6.12(c).

Fig. 6.12(a) ZCS-Buck Converter with M-Type, (b) Simplified equivalent circuit, (c) steady state waveforms

6.4.2.2 The ZCS Boost Converter:

Let us consider the boost-quasi resonant converter with M-type switch as shown in Fig. 6.13(a), with its equivalent circuit is shown in Fig. 6.13(b). Here we assumed the input current is constant, and the load voltage is constant.

Fig. 6.13(a) ZCS converter with Boost-M-Type switch, (b) Simplified equivalent circuit

The following operation assumes half-wave operation.

Mode I \( [0 \leq t < t_1] \):

We first assume that the switch and the diode are both ON in Mode I as shown in Fig. 6.14(a).

Fig. 6.14(a) Equivalent Circuit for Mode I.

The output voltage is given by

\[
V_o = L \frac{di_I}{dt}
\]  \quad (6.34)
The initial inductor current and capacitor voltage are given by,

\[
i_L(0) = 0 \\
v_c(0) = V_o
\]

Integrating Eq. (6.34), the inductor current becomes,

\[
i_L(t) = \frac{V_o}{L} t + i_L(0) = \frac{V_o}{L} t
\]

When the resonant inductor current reaches the input current, \(I_{in}\), the diode turns \(OFF\), hence we have

\[
\frac{V_o}{L} t_1 = I_{in}
\]

with \(t_1\) is given by,

\[
t_1 = \frac{I_{in} L}{V_o}
\]

At \(t = t_1\), the diode turns \(OFF\) since \(i_L = I_{in}\), and the converter enters Mode II. The waveform for \(i_L\) and \(v_c\) are given in Fig. 6.15.

**Mode II \([t_1 \leq t < t_2]\):**

![Fig. 6.14(b) Equivalent circuit for Mode II.](image)

The switch remains close, but the diode is \(OFF\) at \(t_1\) in Mode II as shown in Fig. 6.14(b). This is a resonant Mode during which the capacitor voltage starts decreasing resonantly from its initial value of \(V_o\). When \(i_L = I_{in}\), the capacitor reaches its negative peak. At \(t = t_2\), \(i_L\) equals zero, and the switch turns \(OFF\), hence, switching at zero-current.

The initial conditions are given by,

\[
v_c(t_1) = V_o \quad \text{and} \quad i_L(t_1) = I_o
\]

From Fig. 6.14(b), the first derivatives for \(i_L\) and \(v_c\) are given by,

\[
L \frac{di_L}{dt} = v_c
\]
Next the two first order differential equations need to be solved in this mode. Using the same solution technique used in the buck converter to solve the above differential equations, the expression for \( i_L(t) \) and \( v_c(t) \) are given by,

\[
i_L(t) = I_{in} + \frac{V_o}{Z_o} \sin \omega_o (t - t_1) \quad (6.37)
\]

\[
v_c(t) = V_o \cos \omega_o (t - t_1) \quad (6.38)
\]

where \( \omega_o = \frac{1}{\sqrt{LC}} \).

At \( t = t_2 \), \( i_L(t_2) = 0 \) and the time interval can be obtained from evaluating Eq. (6.37) at \( t = t_2 \), to yield,

\[
(t_2 - t_1) = \frac{1}{\omega_o} \sin^{-1}\left(-\frac{I_{in}Z_o}{V_o}\right) = \frac{1}{\omega_o} [\pi + \sin^{-1}\left(-\frac{I_{in}Z_o}{V_o}\right)]
\]  \quad (6.39)

**Mode III \([t_2 \leq t < t_3]\):**

**Fig. 6.13(c) Equivalent circuit for Mode III.**

Mode III starts at \( t_2 \), and the switch and the diode are both open as shown in Fig. 6.14(c). Since \( v_c \) is constant, the capacitor starts charging up by the input current source. The capacitor voltage is given by,

\[
v_c = \frac{1}{C} \int_{t_2}^{t} I_{in} dt = \frac{I_{in}}{C} (t - t_2) + v_c(t_2) \quad (6.40)
\]

The diode begins conducting at \( t = t_3 \) when the capacitor voltage is equal to the output voltage, i.e. \( v_c(t_3) = V_o \). Equation (6.40) becomes,

\[
V_o = \frac{I_{in}}{C} (t_3 - t_2) + v_c(t_2)
\]

so the time interval in this period in Eq. (6.41)
\[ t_3 - t_2 = \frac{I_{in}}{C} [V_o - v_c(t_2)] \]  

(6.41)

Where \( v_c(t_2) \) may be obtained from Eq. (6.38).

**Mode IV \([t_3 \leq t < t_4]\):**

*Fig. 6.14(d) Equivalent circuit for Mode IV.*

At \( t_3 \), the capacitor voltage is clamped to the output voltage, and the diode starts conducting again, but the switch remains open. This condition remains as long as the switch is open. The cycle of the mode will repeat again at the time of \( T_s \) when \( S \) is turned \textit{ON} again.

Typical steady state waveforms are shown in Fig. 6.15.

*Fig. 6.15 Steady state waveform of the boost-L-Type switch converter*

**Voltage Gain:**

As we did for the buck converter, we apply the conservation of energy per switching cycle to express the voltage gain, \( M = V_o / V_{in} \), in terms of the circuit parameter.

The input energy is given by,

\[ E_{in} = V_{in} I_{in} T_s \]  

(6.42)

and the output energy,

\[ E_o = \int_0^{T_s} i_o V_o \, dt \]  

(6.43)

The output current equals \( i_o = I_{in} - i_L \) and \( i_o = I_{in} \) for intervals \( 0 \leq t \leq t_1 \) and \( t_3 \leq t < T_s \), respectively, therefore, \( E_o \) becomes

\[ E_o = \int_0^{t_1} (I_{in} - i_L) V_o \, dt + \int_{t_3}^{T_s} I_{in} V_o \, dt \]  

(6.44)

The input current is obtained from the conservation of output power as:
\[ I_{in} = \frac{V_{in}^2}{V_{in}R} \]

Substitute for the input current and by evaluating Eq. (6.44), the output energy becomes

\[
E_o = V_o \left( I_{in} t_1 - \frac{V_o}{L} t \right) + I_{in} V_o (T_3 - t_1) \\
= V_o \left( I_{in} t_1 - \frac{v_o}{L} t^2 \right) + I_{in} V_o (T_3 - t_3) 
\]  \hspace{1cm} (6.45)

Substitute for \( t_1 = \frac{I_{in} L}{V_o} \) and the interval \( (T_3 - t_3) = T_3 - [t_1 + (t_2 - t_1) + (t_3 - t_2)] \), with equations for \( (t_2 - t_1) \) and \( (t_3 - t_2) \) are given in Eqs. (6.41) and (6.44), Eq. (6.45) becomes,

\[
E_o = \frac{1}{2} I_{in}^2 L + V_o I_{in} [T_3 - \frac{\alpha}{\omega_o} - \frac{C}{I_{in}} V_o (1 - \cos \alpha)] 
\]  \hspace{1cm} (6.46)

Follow similar steps as in the quasi buck converter, it can be shown that the voltage gain expression is given by,

\[
\frac{M - 1}{M} = \frac{f_{ns}}{2\pi} \left[ \frac{M}{2Q} + \alpha + \frac{Q}{M} (1 - \cos \alpha) \right] 
\]  \hspace{1cm} (6.47)

where, \( \alpha, M, I_o \) and \( f_{ns} \) are given in Eq. (6.29)

Fig. 6.16 shows the characteristic curve for \( M \) vs. \( f_{ns} \) as a function of the normalized load.

**Fig. 6.16 Characteristic curve for \( M \) vs. \( f_{ns} \) for the boost ZCS converter**

### Example 6.2

Design a boost ZCS converter for the following parameters: \( V_{in} = 20V, V_o = 40V, P_o = 20W, f_s = 250kHz. \)

**Solution:**

The voltage gain is \( M = \frac{V_o}{V_{in}} = \frac{40}{20} = 2 \). Let us select \( f_{ns} = 0.38 \). From the characteristic curve of Fig. 6.16, \( Q \) can be approximated to 6.0
The characteristic impedance is given by,
\[ Z_o = \frac{R_o}{Q} = \frac{80\,\Omega}{6} = 13.33\,\Omega \quad (6.48) \]
and the resonant frequency is,
\[ f_o = \frac{f_s}{f_{ns}} = \frac{250\,kHz}{0.38} = 657.89kHz \quad (6.49) \]

Solve Eqs. (6.48) and (6.49) for L and C

\[ L = \frac{Z_o}{2\pi f_o} = \frac{13.33\,\Omega}{2\pi \times 657.89 \times 10^3} = 3.22 \times 10^{-6} \, H \]

\[ C = \frac{1}{Z_o \omega_o} = \frac{1}{(13.33)(2\pi \times 657.89 \times 10^3)} = 18.14 \, nF \]

To limit the input ripple current and the output voltage, we set,

\[ L_o = 100L = 322 \times 10^{-6} \, H \]

\[ C_o = 100C = 1.8 \times 10^{-6} \, F \]

**Example 6.3**

Design a boost converter with ZCS, with the following design parameters: \( V_{in} = 25V \), \( P_0 = 30W \) at \( I_0 = 0.5A \), and \( f_s = 100kHz \). Assume the output voltage ripple \( \Delta V_o \) is 0.2%.

**Solution:**

The load resistance, \( R = \frac{P_0}{I_0^2} = \frac{30}{(0.5)^2} = 120\Omega \)

\[ M = \frac{V_o}{V_s} = \frac{60}{25} = 2.4, \]

From the characteristic curve of Fig. 6.15, we approximate \( Q \approx 6 \) when we assume \( f_n = 0.58 \).

**Hence**, \( f_o = \frac{100}{0.58} = 172.4kHz. \)
From Q and R₀, the characteristic impedance is obtained from,

\[ Q = \frac{R_0}{Z_0} = \frac{120}{11} = 6, \text{ and } Z_0 = 20 \]

hence, \[ \sqrt{\frac{L}{C}} = 20 \]

\[ \omega_0 = 2\pi (172 \times 10^3) = 1080.7 \times 10^{3} \text{ rad / sec} \]

\[ \sqrt{\frac{1}{LC}} = 1080.7 \times 10^{3} \]

Solving for C and L

\[ C = 46.27\eta F \]

\[ L = 18.51\mu F \]

From \( \frac{\Delta V}{V_0} = 0.2\% \), \( C_0 \) can be obtained from the ripple voltage equation for the conventional boost converter, which is given by,

\[ \frac{\Delta V_0}{V_0} = \frac{D}{f_s R_0 C_0} \]

Where DT is the ON time of the switch

\[ t_1 = \frac{L_i i}{V_0} = \frac{18.51 \times 10^{-3} \times 1.2}{60} = 0.370\mu s \]

\[ t_2 - t_1 = \frac{1}{\omega_0} \sin^{-1} \left[ -\frac{Z_0 i}{V_0} \right] \]

\[ = \frac{1}{1080.7 \times 10^{3}} \sin^{-1} \left[ -\frac{20 \times 1.2}{60} \right] = 3.29\mu s \]

\[ t_3 - t_2 = \frac{V_0}{\omega_0 Z_0 i} (1 - \cos \alpha) \]

\[ = \frac{1}{1080.7 \times 10^{3}} \frac{60}{20 \times 1.2} (1 - \cos 3.553) \]

\[ = 0.193\mu s \]

\[ t_4 - t_3 = T - t_1 - (t_{i2}) - (t_{23}) \]

\[ = 10 - 0.370 - 3.29 - 0.193 = 6.147\mu s \]
Figure 6.17(a) shows the quasi-resonant boost converter by using the L-type resonant switch, and the simplified circuit and its steady state waveforms are shown in Fig.6. 17(b) and (c), respectively

Fig. 6.17(a) ZCS boost converter with-L-Type Switch, (b) Simplified equivalent circuit, (c) steady state waveforms

The reader is invited to verify these waveforms.

Exercise 6.3

Consider a boost quasi resonant converter of Fig. (6.13) with the following design parameters: \( V_{in}=25V \), \( P_0=30W \) at \( I_0=0.5A \), and \( f_s=100kHz \). Determine the resonant tank capacitor and inductor components.

Ans. \( C=46.30 \mu F \) and \( L=18.5 \mu H \)

6.4.2.3 ZCS Buck-boost Converter

Let us consider the quasi-resonant buck/boost converter by using the L-type switch as shown in Fig. 6.18(a), and (b) is the simplified equivalent circuit.

Fig. 6.18(a) ZCS Buck-boost with L-Type switch, (b)Simplified equivalent circuit.

Like the buck and the boost converters, the buck-boost converter also leads to four modes of operations.

Mode I \([ 0 \leq t < t_1 ]\):

Fig. 6.19(a) Equivalent Circuit for Mode I.

Mode I starts at \( t = 0 \), the switch and the diode are both conducting. According to the Kirchhoff’s law, the voltage equation can be written as
\[ \frac{L}{dt} \frac{di_L(t)}{dt} = V_{in} + V_o \]  

(6.50)

By integrate both sides of Eq. (6.50) with the initial condition of \( i_L(0) = 0 \), \( i_L(t) \) is given by,

\[ i_L(t) = \frac{V_{in} + V_o}{L} t \]  

(6.51)

and \( v_c(t) = -V_o \)

At \( t = t_1 \), the inductor current reaches the input current, forcing the output diode stop conducting, so \( t_1 \) can be express as

\[ t_1 = \frac{LI_{in}}{V_{in} + V_o} \]  

(6.52)

The steady state waveforms are shown in Fig. 6.20.

**Mode II \([t_1 \leq t < t_2]\):**

*Fig. 6.19(b) Equivalent circuit for Mode II.*

This is a resonant stage between \( L \) and \( C \) with the initial conditions given by,

\[ v_c(t_1) = 0 \]
\[ i_L(t_1) = I_F \]

According the Kirchhoff’s law, from the Fig. 6.19(b), the equation can be given as

\[ \frac{L}{dt} \frac{di_L}{dt} = V_{in} + V_o - v_c \]  

(6.53a)

\[ C \frac{dv_c}{dt} = -i_L + I_F \]  

(6.53b)

Solving Eqs. (6.53) for \( t > t_1 \), we obtain,

\[ i_L(t) = I_F + \frac{V_{in} + V_o}{Z_o} \sin \omega_o (t - t_1) \]  

(6.54)

\[ v_c(t) = (V_{in} + V_o)[1 - \cos \omega_o (t - t_1)] \]  

(6.55)
at \( t = t_2 \), the inductor current reaches zero, \( i_L(t_2) = 0 \), and the switch stops conducting. The time interval \((t_2 - t_1)\) is given by,

\[
(t_2 - t_1) = \frac{1}{\omega_o} \sin^{-1}\left(-\frac{I_Z}{V_{in} + V_o}\right) \tag{6.56}
\]

**Mode III \([t_2 \leq t < t_3]\):**

*Fig. 6.19(c) Equivalent circuit for Mode III*

Mode III starts at \( t = t_2 \) when the inductor current reaches zero. The switch and the diode are both OFF. The capacitor starts to discharge until it reach zero, and the diode will start to conduct again at \( t = t_3 \). During this period, the inductor current is zero.

\[
v_c = -\frac{1}{C} \int_{t_2}^t I_T dt = -\frac{I_F}{C}(t - t_2) + v_c(t_2) \tag{6.57}
\]

The diode begins to conducting at the end of this mode, \( t = t_3 \), because the capacitor voltage is equal to zero.

\[
0 = -\frac{I_F}{C}(t_3 - t_2) + v_c(t_2)
\]

where \( v_c(t_2) = V_{in} - (V_{in} - V_o) \cos\alpha \) obtained from the previous mode. The expression from Eq. (6.57) for the time between \( t_2 \) and \( t_3 \) is,

\[
(t_3 - t_2) = \frac{C}{I_F}v_c(t_2) \tag{6.58}
\]

**Mode IV \([t_3 \leq t < t_4]\):**

*Fig. 6.19(d) Equivalent circuit for Mode IV*

Between \( t_3 \) and \( t_4 \), the switch remains OFF, but the diode is ON. At the end of the cycle, switch is close again when the current is zero. The cycle of the modes will repeat again at the \( T_s \).

The steady state waveform is shown in Fig. 6.20 are the characteristic waveforms for the switch, \( v_c \), and \( i_L \).

*Fig. 6.20(a) Steady state Waveform of buck/boost L-Type Switch*
Voltage Gain:

As before, the conservation of energy per switching cycle is used to express the voltage gain, \( M = V_o / V_i \), in term of the normalized circuit parameter. It can be shown that M for the buck-boost-ZCS converter is given by

\[
M = \frac{f_{ns}}{1 + M} = \frac{f_{ns}}{2\pi} \left\{ \frac{M}{2Q} + \alpha + \frac{Q}{M} [1 - \cos(\alpha)] \right\}
\]  

(6.59)

Figure 6.21 shows the characteristic curve for M vs. fns for the ZCS buck-boost converter.

Fig. 6.21 characteristic curve for M vs. fns for the ZCS buck-boost converter.

Example 6.4

Consider a buck-boost QRC-ZCS converter with the following specifications: \( V_{in} = 40V \), \( P_o = 80W \) at \( I_o = 4A \), \( f_s = 250kHz \), \( L_o = 0.1mH \), and \( C_o = 6 \mu F \). Design values for L and C and determine the output ripple voltage.

Solution:

The output voltage and load resistance are given by,

\[
V_o = \frac{80}{4} = 20V \quad \text{and} \quad R_o = \frac{20}{4} = 5\Omega
\]

The voltage gain is given by,

\[
\frac{V_o}{V_s} = \frac{20}{40} = 0.5
\]

With \( M = 0.5 \), and \( f_{ns} = 0.17 \), we select \( Q = 3 \),

Resulting in \( f_o \) equals to \( f_o = \frac{250}{0.17} = 1470.6kHz \)

From Q, and \( Z_o \), we have

\[
Q = \frac{R_o}{Z_o} = \frac{5}{Z_o} \quad \text{and} \quad \frac{Z_o}{Z_o} = \frac{3}{5} = 0.6\Omega \]. Hence,

\[
\sqrt{\frac{L}{C}} = 0.6
\]

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and,
\[
\sqrt{1/LC} = 2\pi f_o = 2\pi \times 1470.6 \times 10^3
\]
\[
\frac{1}{C} = 0.6 \times 2\pi \times 1470.6 \times 10^3
\]

From the above equation, C and L are given by,
\[
C = 180.4 nF
\]
\[
L = 3^2 \times C_r = 1.6 \mu H
\]

The voltage ripple is,
\[
\frac{\Delta V_o}{V_o} = \frac{D}{RC_o f} = \frac{0.5}{5 \times 6 \times 10^{-6} \times 250 \times 10^3} = 6.67\%
\]

**Exercise 6.4:**

The buck-boost converter with M-type switch as shown in Fig. 6.22(a), and (b) is the equivalent circuit. Derive the steady state waveforms and show that the voltage gain, \( M = \frac{V_o}{V_{in}} \), is given by,
\[
\frac{M}{1 + M} = \frac{f_m}{2\pi} \left\{ \frac{M}{2Q} + \frac{Q}{M} [1 - \cos(\alpha)] \right\}
\]

\[ \text{Eq. 6.60} \]

*Fig. 6.22(a) ZCS Buck-Boost Converter with-M-Type Switch, (b) Simplified equivalent circuit*

### 6.5 ZERO-VOLTAGE SWITCHING TOPOLOGIES

In this section, we will investigate the Zero-Voltage-Switching (ZVS) Quasi-resonant converter family. Like the ZCS topologies, M-type or L-type switch arrangements can be used. In those topologies, the power switch is turned ON at zero-voltage (of course the turn OFF also occurs at zero-voltage). While the switch is OFF, a peak voltage will appear across it, causing the stress to be higher than those in the hard-switching PWM case. In a ZVS topology, a flyback diode across the switch (body diode) is used to damp the voltage across the capacitor, which in turns results in a zero-voltage across the switch. We should point out that the capacitor across the switch can be the same as the switch
parasitic capacitor, and the fly-back diode could also be the same as the internal body
diode of the power semiconductor switch.

Figure 6.23(a) shows a MOSFET switch implementation by including internal body
diode and parasitic capacitance.

![Fig. 6.23(a) MOSFET Implementation, (b) MOSFET switch with fast flyback diode.](image)

We will assume $C_{gd}$ and $C_{gs}$ are too small to be included. If the body diode is not fast
enough for the designed application or has limited power capabilities, it is practically
possible to block it and use external, fast flyback diode as shown in Fig. 6.23(b). $D_1$ is
used to block $D_s$ and $D_2$ is the actual diode used to carry the reverse switch current.
Both the current and voltage-mode control methods are used in conjunction with direct
duty ratio PWM control approach to vary the $ON$ or $OFF$ time of the power switch.

Over the last fifteen years, many different resonant converter topologies have been
introduced. Only soft switching ZVS topologies will be analyzed and their control
characteristic curves will be studied. Regardless of their topological variations, many of
these converters have several common features to be discussed next.

6.5.1 Resonant Switch Arrangements

Next we investigate the buck, boost and buck-boost ZVS topologies using the L-type and
M-type resonant switches. Figure 6.24(a) show the two possible switch implementation
using L- and M-type resonant switches. The half-wave L-type and M-type MOSFET
implementation are shown in Fig 6.24(b). Whereas, Fig 6.24(c) show the full-wave
implementations for L- and M-type switches.

![Fig. 6.24 (a) Resonant switch arrangement types for ZVS operation (b) Half-Wave MOSFET Implementation (c) Full-Wave MOSFET Implementation](image)

6.5.2 Steady State Analyses

Like in the ZCS case, to simplify the steady state analysis, we make here the same
assumptions made in 6.4.2.

6.5.2.1 The Buck Converter

Replacing the switch in Fig. 6.7(a) by the M-type switch of Fig. 6.24(a), we obtain a new
ZVS buck converter as shown in Fig. 6.25(a) is obtained. The simplified equivalent
circuit is given in Fig. 6.25(b).
As the switch and diode are ON and/or OFF at the same time, it can be shown that under steady-state conditions, there are four modes of operation. Unlike ZCS topologies, the switching cycle in ZVS starts with the main switch first being in the conduction state. This is because in order to establish a zero-voltage condition across the switch during the resonant stage while it is open.

**Mode I [ 0 ≤ t < t₁ ]:**

Assume initially the power switch is conducting, and the diode is OFF. Mode I starts at t=0 when the switch is turned OFF. In this mode, since S has been closed, the initial capacitor voltage, \( v_c \) is zero, and the inductor current is \( I_o \) as shown in Fig. 6.26(a).

\[
\begin{align*}
  v_c(0) & = 0 \\
  i_L(0) & = I_o \\
\end{align*}
\]

Applying KVL to Fig. 6.26(a), we have,

\[
C \frac{dv_c}{dt} = i_L
\]

since \( i_L = I_o \), the capacitor start to charge according to Eq. (6.61),

\[
v_c = \frac{1}{C} I_o t
\]  \( (6.61) \)

The voltage across the output diode is given by,

\[
v_D = V_{in} - v_c
\]

As long as \( v_c < V_{in} \), the diode remains OFF.

The capacitor voltage reaches the input voltage, \( V_{in} \), at \( t=t_1 \), causing the diode to turn ON. Hence, at \( t=t_1 \), we have

\[
v_c(t_1) = V_{in}
\]

and \( t_1 \) can be expressed as
At \( t = t_1 \), the circuit enters Mode II. The current and voltage waveforms are shown in Fig. 6.27.

Fig. 6.27 Steady state waveforms for the ZVS Buck Converter

**Mode II \([t_1 \leq t < t_2]\):**

Mode II starts at \( t_1 \) when the diode turns ON, and the circuit enters the resonant stage. During the time between \( t_1 \) and \( t_2 \), the switch remains OFF. At \( t = t_2 \), the capacitor voltage tends to go negative, hence, forcing the diode across S to turn ON. The initial capacitor voltage and inductor current in this mode are given by,

\[
v_c(t_1) = V_{in} \\
i_L(t_1) = I_o
\]

The expressions of the current and the voltage in time domain are, given in Eq. (6.63) and (6.64), respectively

\[
i(t) = I_o \cos \omega_o (t - t_1) \quad \text{(6.63)}\\
v_c(t) = V_{in} + I_o Z_o \sin \omega_o (t - t_1) \quad \text{(6.64)}
\]

where, the resonant frequency and the characteristic impedance are defined as before.

The inductor current is zero when the capacitor voltage reaches the peak, and the capacitor starts discharging while the inductor current is negative value. The inductor current reaches the peak when the capacitor drops to the input voltage, and at the end of the mode, \( t = t_2 \), the capacitor equals to zero, \( v_c(t_2) = 0 \)

The period between \( t_2 \) and \( t_1 \) is given by,

\[
t_2 - t_1 = \frac{1}{\omega_o} \sin^{-1}\left(\frac{-V_{in}}{I_o Z_o}\right) \quad \text{(6.65)}
\]

\[
= \frac{\alpha}{\omega_o}
\]

and the inductor current at \( t = t_2 \) is,

\[
i_L(t_2) = I_o \cos \alpha
\]
where $\alpha = \sin^{-1}\left(\frac{-V_{\text{in}}}{I_o Z_o}\right)$

**Mode III $[t_2 \leq t < t_3 ]$:**

At $t_2$, the capacitor voltage becomes zero, and the inductor current starts to charge linearly, and reaches the output current at $t = t_3$. The body diode of the switch turns $\text{ON}$, and the output diode also remains on at this point as shown in Fig. 6.26(c). As long as the inductor current is less than $I_o$, the output diode will stay $\text{ON}$.

The initial value of capacitor voltage in Mode III is zero

$$v_c(t_2) = 0$$

The inductor voltage is equal to the input voltage,

$$L \frac{di_L}{dt} = V_{\text{in}}$$ \hspace{1cm} (6.67)

integrating Eq. (6.67) from $t_2$ to $t$, the inductor current can be expressed as,

$$i_L(t) = \frac{V_{\text{in}}}{L} (t - t_2) + I_o \cos \alpha$$ \hspace{1cm} (6.68)

At $t = t_3$, the inductor current reaches the output current $i_L(t_3) = I_o$ forcing the diode to turn $\text{OFF}$.

Hence, the time interval form $t_3$ to $t_2$ is

$$t_3 - t_2 = \frac{I_o L}{V_{\text{in}}} (1 - \cos \alpha)$$ \hspace{1cm} (6.69)

Therefore, there will be no power transfer and no charge or discharge interval remains when the switch turns on.

**Mode IV $[t_3 \leq t < t_4 ]$:**
At this mode the switch (body diode) remains on, but the output diode stops conducting at $t = t_3$. Mode IV will continue as long as the switch is on. The inductor current is equal to the output current, and the capacitor voltage is zero at this stage.

\[
i_L = I_o,
\]
\[
v_c = 0
\]

By turning off the switch at $t = t_4 = T_s$, the switching cycle repeats. The dead time $t_4 - t_3$ is given by,

\[
t_4 - t_3 = T_s - \Delta t_1 - \Delta t_2 - \Delta t_3
\]  \hfill (6.70)

**Voltage Gain:**

We follow the same approach as in the ZCS by using the energy balance concept. The input energy is given by,

\[
E_{in} = \int_0^{T_s} i_{in} V_{in} \, dt
\]

$i_{in}$ is the current which is equal to $i_L(t)$. Hence, we have

\[
E_{in} = \int_0^{t_1} i_L(t) V_{in} \, dt + \int_{t_1}^{t_2} i_L(t) V_{in} \, dt + \int_{t_2}^{t_3} i_L(t) V_{in} \, dt + \int_{t_3}^{T_s} i_L(t) V_{in} \, dt
\]  \hfill (6.71)

The inductor current equals to the output current in Mode I and IV, and for $t_1 \leq t < t_2$ and $t_2 \leq t < t_3$, $i_L$ is given in Eqs. (6.63) and (6.68), respectively. Substitute the inductor currents into Eq. (6.71) to yield Eq. (6.72),

\[
E_{in} = V_{in} [I_o t_1 + I_o \sqrt{L/C} \sin \omega_o (t_2 - t_1) + \frac{V_{in}}{2L} (t_3 - t_2)^2
\]
\[
+ I_o \cos \alpha (t_3 - t_2) + I_o (T_s - t_3)]
\]  \hfill (6.72)

Substituting for the time intervals $t_1$, $(t_2-t_1)$, $(t_3-t_2)$ and $(T_s-t_3)$ from Eqs. (6.62), (6.65), (6.69) and (6.70), respectively, and using the normalized parameters $M, Q, \omega_o$ Equation (6.72) becomes,

\[
E_{in} = V_{in} I_o \left[ \frac{-Q}{M \omega_o} - \frac{ML}{2R} + T_s - \frac{\alpha}{\omega_o} + \frac{ML}{R} \cos \alpha - \frac{ML}{2R} \cos^2 \alpha \right]
\]  \hfill (6.73)

The output energy is expressed by,
Equate the input and output energy in Eqs. (6.73) and (6.74) the voltage gain expression becomes,

\[
\frac{V_o}{V_{in}} = 1 - \frac{f_{ns}}{2\pi} \left[ \frac{M}{2Q} + \alpha + \frac{M}{Q} (1 - \cos \alpha) \right]
\]  

(6.75)

A plot of the control characteristics curve of M vs. \( f_{ns} \) is shown in Fig. 6.28.

---

**6.5.3.2. The Boost Converter**

In the following section we consider the quasi-resonant boost converter by using the M-type switch as shown in Fig. 6.29(a) with its simplified circuit shown in Fig. 6.29(b)

*Fig. 6.29(a) Quasi-Resonant Boost Converter with M-Type Switch, (b) Equivalent circuit*

The four circuit modes of operation are discussed as follows:

**Mode I \[ 0 \leq t < t_1 \]:**

*Fig. 6.30(a) Equivalent Circuit Mode I, (b) Mode II, (c) Mode III and (d) Mode IV*

*Fig 6.31 Steady State waveforms For ZVS Boost Converter*

Assume for \( t<0 \), the switch is closed while D is open. At \( t=0 \), the switch is turned \textit{OFF}, making the capacitor to charge by the constant current through it which is given by,

\[
I_{in} = I_L = I_C = C \frac{dv_c}{dt}
\]  

(6.76)

with the initial capacitor voltage equals zero, Eq. (6.76) gives the following expression for \( v_c(t) \):

\[
v_c(t) = \frac{I_{in}}{C} t
\]  

(6.77)
The capacitor voltage reaches the output voltage at \( t = t_1 \), \( v_c(t_1) = V_o \), resulting \( t_1 \) equals to,
\[
t_1 = \frac{CV_o}{I_{in}} \tag{6.78}
\]

At \( t = t_1 \), the diode starts conducting since \( v_c = V_o \), and the converter enters Mode II.

**Mode II \([t_1 \leq t < t_2] \):**

At \( t=t_1 \), the resonant stage begins since D is \( ON \) and S is \( OFF \) as shown in Fig. 6.30(b). When the capacitor voltage reaches the output voltage \( i_L \) reaches the negative peak.

The initial conditions are \( v_c(t_1) = V_o \) and \( i_L(t_1) = I_{in} \).

The expression for \( v_c(t) \) is given by Eq. (6.79),
\[
v_c(t) = V_o + I_{in}Z_o \sin \omega_o(t - t_1) \tag{6.79}
\]

and the inductor current is,
\[
i_L(t) = I_{in}[1 + \cos \omega_o(t - t_1)] \tag{6.80}
\]

Evaluating Eq. (6.79) at \( t=t_2 \) with \( v_c(t_2)=0 \), the time interval between \( t_1 \) to \( t_2 \) is given by,
\[
(t_2 - t_1) = \frac{1}{\omega_o} \sin^{-1}\left(\frac{-V_o}{I_{in}Z_o}\right) \tag{6.81}
\]

**Mode III \([t_2 \leq t < t_3] \):**

Mode III starts at \( t_2 \) when \( v_c \) reaches zero and S turns \( ON \) at \( zvc \). The switch and the diode are both conducting, and the inductor current linearly increases to \( I_{in} \) as shown in Fig. 6.31. At \( t = t_3 \), the diode (anti-parallel diode) turns \( ON \), clamping C.

The initial conditions at \( t = t_2 \) are,
\[
v_c(t_2) = 0 \tag{6.82a}
\]
\[
i_L(t_2) = I_{in}(1 + \cos \omega_o(t_2 - t_1)) \tag{6.82b}
\]
Because the capacitor voltage is zero, the inductor voltage is equal to the output voltage.

\[ L \frac{di_L}{dt} = V_o \]  \hspace{1cm} (6.83)

By taking integration of Eq. (6.83), the inductor current becomes,

\[ i_L(t) = \frac{V_o}{L} (t - t_2) + i_L(t_2) \]  \hspace{1cm} (6.84)

At \( t = t_3 \), \( i_L \) reaches zero, resulting in the time interval to be given in Eq. (6.85)

\[ (t_3 - t_2) = \frac{L}{V_o} i_L(t_2) \]  \hspace{1cm} (6.85)

Substituted the initial condition into the equation, we get,

\[ (t_3 - t_2) = \frac{L}{V_o} I_{in} (1 + \cos \omega_o (t_2 - t_1)) \]  \hspace{1cm} (6.86)

At \( t = t_3 \), the output diode turns OFF and the entire \( I_{in} \) current flows in the transistor and the inductor.

**Mode IV \( [t_3 \leq t < t_4] \):**

At the time \( t_3 \), the inductor current reaches zero, the output diode turns OFF, but the switch remains close. The cycle of the mode will repeat again at \( T_s \).

**Voltage Gain:**

As before, we use the conservation of energy per switching cycle to express the voltage gain. It can be that the voltage gain in terms of the normalized parameter is given in Eq. (7.87).

\[ \frac{1 - M}{M} = \frac{f_{ns}}{2\pi} \left[ \frac{M}{2Q} + \alpha + \frac{M}{Q} (1 - \cos \alpha) \right] \]  \hspace{1cm} (7.87)

*Fig. 6.32 M vs. fns for ZVT boost converter*

**Example 6.5**
Design a ZVS-QRC boost converter for the following design parameters: \( V_{\text{in}} = 30\text{V}, \) \( P_0 = 30\text{W} \) at \( V_0 = 38\text{V}, f_{\text{ns}} = 0.4, \) and \( T_s = 4\mu\text{s}. \) Assume output voltage ripple is limited to 2%.

**Solution:**

The voltage gain is \( M = \frac{V_o}{V_{\text{in}}} = 1.3. \) With \( f_{\text{ns}} = 0.4 \) and \( M = 1.3, \) we obtain \( Q = 0.2. \)

The switching frequency \( f_s = \frac{1}{T_s} = \frac{1}{4\mu\text{s}} = 250kHz \)

resulting in

\[
f_o = \frac{f_s}{0.4} = \frac{250}{0.4} = 625kHz
\]

\[
\therefore \frac{1}{\sqrt{L/C}} = (2\pi)(625) \times 10^3
\]

\[
Q = \frac{R_o}{Z_o} = 0.2
\]

\[
\frac{R_o}{\sqrt{L/C}} = 0.2
\]

\[
R_o = \frac{38^2}{30} = 48.13\Omega
\]

\[
\therefore \frac{1}{\sqrt{L/C}} = \frac{0.2}{48.13} \quad \text{or} \quad \sqrt{\frac{L}{C}} = \frac{48.13}{0.2} = 240.65
\]

Solving for \( C \) and \( L, \)

\[
C = \frac{1}{(2\pi)(625)(10^3)(240.65)} = 1.06nF
\]

\[
L = 1.06 \times 10^{-9} \times (240.65)^2 = 61.3\mu F
\]

To calculate \( L_o \) and \( C_o, \) using the voltage ripple 0.2%,

\[
\therefore 0.2\% = \frac{D}{f_sR_oC_o}
\]

\[
D = \frac{f_a}{f_s} = 0.4
\]

\[
0.2 \times \frac{0.4}{100} = \frac{D}{f_sR_oC_o}
\]

\[
C_o = \frac{0.4 \times 100}{f_sR_o \times 0.2}
\]
To achieve limited ripple current, it is recommended that $L_o$ be set to about 100 times the critical inductor value. So we select $L_o=140 \, \mu H$.

---

**Exercise 6.5:**
The quasi-resonant boost converter by using the L-type switch as shown in Fig. 6.33(a) and its simplified circuit is shown in Fig. 6.33(b)

*Fig. 6.33 (a) Quasi-Resonant Boost Converter with L-Type Switch and (b) Simplified equivalent circuit*

Derive the expression for the voltage gain.

---

### 6.5.3.3 The Buck-Boost Converter

The ZVS buck-boost converter with M-type switch is shown in Fig. 6.34(a) with its equivalent circuit shown in Fig. 6.34(b).

*Fig. 6.34 (a) ZVS Buck-Boost-M-Type, (b) Simplified equivalent Circuit*

Like the buck and the boost converters, the buck/boost converter also leads to four modes of operations as shown in Fig. 6.35. Figure 6.35(e) shows typical steady state waveforms for $v_c$ and $i_L$.

*Fig. 6.35 Equivalent circuit for Mode I, (b) Mode II, (c) Mode III, (d) Mode IV, (e) Steady state waveforms for $v_c$ and $i_L$*

Following similar analysis as before, it can be shown that the voltage gain in term of $M$, $Q$, and $f_{ns}$ is given by Eq. (6.88),

$$M = \frac{1}{\frac{f_{ns}}{2\pi} \left[ \alpha + \frac{M}{2Q} + \frac{M}{Q} (1 - \cos \alpha) \right]} - 1 \quad (6.88)$$

Figure 6.36 show the control characteristic curve for $M$ vs. $f_{ns}$. 

---
6.6 GENERALIZED ANALYSIS FOR ZCS*

It can be shown from the above analysis of quasi resonant ZCS and ZVS PWM converters that each dc-dc converter family shares the same switching network, with orientation that depends on the topology type (Buck, Boost, Buck-Boost, Cuck, Zeta, SEPIC, …etc.). As a result, the switching network representation and analysis, including the switching waveforms, can be generalized for each converters family.

The generalized analysis means that in order to derive the analysis and the design curves for a family of converters, only the generalized switching-cell with generalized and normalized parameters for that family need to be analyzed. The resultant equations from the generalized cell will be the generalized equations that describe any converter that uses this specific cell. By using generalized parameters, it is possible to generate a single transformation table from which the voltage converter ratios and other important design parameters for each converter can be obtained directly.

6.6.1 The Generalized Switching-Cell

Figure 6.37(a) and (b) show the generalized switching-cells of the quasi resonant PWM ZCS and ZVS converters, respectively. It can be noted that these cells are all of the common ground three-terminal two-port network type. The following parameters, which will be used throughout this discussion, are defined as follows:

- The normalized cell input voltage ($V_{ng}$):
  \[ V_{ng} = \frac{V_g}{V_{in}} \]
  where $V_g$ is the switching-cell average input voltage as shown in Figure 6.37.

- The normalized cell output current ($I_{nf}$):
  \[ I_{nf} = \frac{I_F}{I_o} \]
  where $I_F$ is the switching-cell average output current.

- The normalized filter capacitor voltage ($V_{nf}$):
  \[ V_{nf} = \frac{V_F}{V_{in}} \]
  where $V_F$ is the filter capacitor average voltage.

* This section can be skipped without lose of continuity.
• The normalized filter inductor current \( I_{nT} \):

\[
I_{nT} = \frac{I_T}{I_o}
\]

Where \( I_T \) is the filter inductor average current (in the ZCS-QSW CC family).

• The normalized cell output average voltage \( V_{nbc} \):

\[
V_{nbc} = \frac{V_{bc}}{V_{in}}
\]

where \( V_{bc} \) is the switching-cell average output voltage.

• The normalized current entering node \( b \) in the switching cell \( I_{nb} \):

\[
I_{nb} = \frac{I_b}{I_o}
\]

where \( I_b \) is the average current entering node \( b \).

Note that the **Generalized Transformation Table** that will be derived later will include the generalized parameters \( V_{ng}, I_{nF}, V_{nbc}, I_{nb}, V_{nF} \) and \( I_{nT} \) which are the normalized versions of the parameters \( V_g, I_F, V_{bc}, I_b, V_F, \) and \( I_T \). It will be noted that \( I_{nb}, V_{nF} \), and \( I_{nT} \) have the same normalized quantity and so for \( V_{ng} \) and \( I_{nF} \).

**Fig. 6.37 Switching-Cells: (a) ZCS-QRC Cell, and (b) ZVS-QRC Cell**

### 6.6.2 The Generalized Transformation Table

The derivation if the Generalized Transformation Table for the converters families is out the scope of this book. The Generalized Transformation Table is shown in Table 6.1.

<table>
<thead>
<tr>
<th></th>
<th>( V_{ng}, I_{nF} )</th>
<th>( V_{nF}, I_{nT}, I_{nb} )</th>
<th>( V_{nbc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>1</td>
<td>1-M</td>
<td>-M</td>
</tr>
<tr>
<td>Boost</td>
<td>M</td>
<td>1</td>
<td>1-M</td>
</tr>
<tr>
<td>Buck-Boost, Cuk, Zeta, and Sepic</td>
<td>1+M</td>
<td>1</td>
<td>-M</td>
</tr>
</tbody>
</table>

By applying this cell to the conventional DC/DC converters, the ZCT-QRC family can be formed as shown in Fig. 6.38.

**Fig. 6.38 The DC/DC ZCS-QRC Family:**
(a) Buck (b) Boost (c) Buck-Boost (d) Cuk (e) Zeta. (f) SEPIC.
In the next section, the modes of operation for the ZVS-QRC switching network will be discussed briefly and the main switching waveforms will be drawn in terms of the generalized parameters.

6.6.3 The Basic Operation of the ZCS-QRC Cell

The typical switching waveforms for the cell in Fig. 6.37(a) are shown in Fig. 6.39. Table 6.2 shows the condition of the switches and diodes in each mode. It can be shown that there are four modes of operation and their analysis is summarized as follows:

Mode 1 \((t_o \leq t \leq t_1)\) It is assumed that before \(t = t_0\), \(S\) was \(OFF\) and \(D\) was \(ON\) to cury \(I_F\). The resonant inductor \(L\) was currying no current and the resonant capacitor \(C\) voltage was zero. Mode 1 starts when \(S\) is turned \(ON\) while \(D\) is \(ON\), which cause \(L\) to charge up linearly until the current through it becomes equal to \(I_F\) at \(t = t_1\) causing \(D\) to Turn \(OFF\).

Mode 2 \((t_1 \leq t \leq t_2)\) starts when \(D\) turns \(OFF\) while \(S\) is \(ON\) causing a resonant stage between \(C\) and \(L\) to start until the current through \(L\) drops to zero at \(t = t_2\) causing \(S\) to turn \(OFF\) at zero current (Soft-switching).

Mode 3 \((t_2 \leq t \leq t_3)\) starts when \(S\) is turns \(OFF\) at zero. The resonant capacitor voltage starts discharging linearly until it drops to zero again causing \(D\) to turn \(ON\) at zero voltage at \(t = t_3\).

Mode 4 \((t_3 \leq t \leq t_o + T_s)\) is a steady-state mode and nothing happen on it until \(S\) is turned \(ON\) again to start the next switching cycle.

Generalized Steady State Analysis:

From the description of the modes of operation in the last Section, the equivalent circuit for each mode can be drawn as shown in Fig. 6.40. These equivalent circuits can be used along with the description of the modes to write the mathematical equations for each mode as follows (knowing that \(v_c(t_o) = 0\) and \(i_r(t_o) = 0\)):

![Fig. 6.39: Main ZCS-QRC switching cell waveforms](image-url)

**Table 6.2:** Switches conditions

<table>
<thead>
<tr>
<th>Mode ((t_o \leq t \leq t_1))</th>
<th>S</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 ((t_o \leq t \leq t_1))</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>Mode 2 ((t_1 \leq t \leq t_2))</td>
<td>ON</td>
<td>OFF</td>
</tr>
<tr>
<td>Mode 3 ((t_2 \leq t \leq t_3))</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>Mode 5 ((t_3 \leq t \leq t_o + T_s))</td>
<td>OFF</td>
<td>ON</td>
</tr>
</tbody>
</table>
Mode 1: \( t_o \leq t \leq t_1 \):
\[
v_c(t) = 0
\]
\[
i_i(t) = \frac{V_g}{L}(t-t_o)
\]
\[
v_c(t_1) = 0
\]
\[
i_i(t_1) = I_F
\]  
(6.89)

Mode 2: \( t_1 \leq t \leq t_2 \)
\[
v_c(t) = V_g [1 - \cos \omega_o(t-t_1)]
\]  
(6.91)
\[
i_i(t) = I_F + \frac{V_g}{Z_o} \sin \omega_o(t-t_1)
\]  
(6.92)
\[
i_L(t_2) = 0
\]  
(6.93)

Mode 3: \( t_2 \leq t \leq t_3 \)
\[
v_c(t) = -\frac{I_F}{C}(t-t_2) + V_g [1 - \cos \beta]
\]  
(6.94)
\[
i_i(t) = 0
\]
\[
v_c(t_3) = 0
\]  
(6.95)

Mode 4: \( t_3 \leq t \leq t_o + T_s \)
\[
v_c(t) = 0
\]
\[
i_L(t) = 0
\]

Fig. 6.40: The equivalent circuits for
(a) Mode 1, (b) Mode 2, (c) Mode 3, and (d) Mode 4

Generalized Intervals Equations

To simplify the analysis, the following time intervals are defined:
\[
\alpha = \omega_o(t_1 - t_o)
\]
\[
\beta = \omega_o(t_2 - t_1)
\]
\[
\gamma = \omega_o(t_3 - t_2)
\]
\[
\delta = \omega_o((t_0 + T_s) - t_3)
\]

These intervals can be derived as follows:

- From Equations (6.89) and (6.90):
\[ \alpha = \omega_o (t_1 - t_0) = \frac{Z_o I_F}{V_s} \]

By using the normalized parameters, we have
\[ \alpha = \omega_o (t_1 - t_0) = \frac{MI_{nF}}{QV_{ng}} \quad (6.96) \]

- From Equations (6.92) and (6.93); \( \beta \) is given by,
\[ \beta = \omega_o (t_2 - t_1) = \sin^{-1}(-\frac{MI_{nF}}{QV_{ng}}) \quad (6.97) \]

- From Equations (6.94) and (6.95):
\[ \gamma = \omega_o (t_3 - t_2) = \frac{QV_{ng}}{MI_{nF}} (1 - \cos \beta) \]

- From Fig. 6.39 and intervals \( \alpha, \beta \) and \( \gamma \), we have \( \delta \) given by,
\[ \delta = \omega_o ((t_0 + T_s) - t_3) = \frac{2\pi}{f_{ns}} - \alpha - \beta - \gamma \quad (6.98) \]

**Generalized Gain Equation**

The cell output to input generalized gain can be found using the average output diode D voltage as follows:
\[ V_{D-ave} = -V_{C-ave} \]
\[ = -\frac{1}{T_s} \int_{t_0}^{t_c+T_s} v_c(t) \, dt \]
\[ = -\frac{1}{T_s} \left[ V_g ((t_2 - t_1) - \frac{\sin \beta}{\omega_o} - \frac{I_F}{2C} (t_3 - t_2)^2 + V_g (1 - \cos \beta)(t_3 - t_2) \right] \]

By using the normalized parameters defined previously, we have:
\[ V_{nD} = \frac{f_{ns}}{2\pi} \left[ \frac{MI_{nF}}{2Q} \gamma^2 - V_{ng} (\beta + \gamma - \sin \beta \cos \beta) \right] \quad (6.99) \]

By substituting for the generalized parameters \( V_{nD}, V_{ng}, \) and \( I_{nF} \) from Table 6.1 in Equation (6.95), we will have the gain equation for each converter in the family.
Generalized ZCS Condition

It can be noted from Fig. 6.39 that \( S \) can be turned OFF at anytime after \( t = t_2 \). The generalized condition to achieve zero-current switching can be expressed as follows:

\[
\frac{2\pi}{f_{ns}} (D) \geq \alpha + \beta
\]

(6.100)

Generalized Peak Resonant Inductor Current (Peak Switch Current)

The peak resonant inductor current or peak switch current occurs at \( t = t_{L-p} \) when \( \omega_o (t_{L-p} - t_1) = \pi / 2 \). By using Equation (6.92) at \( t = t_{L-p} \):

\[
I_{n,L-p} = I_{nf} + \frac{QV_{ng}}{M}
\]

(6.101)

The peak resonant capacitor voltage or peak diode voltage occurs at \( t = t_{C-p} \) when \( \omega_o (t_{C-p} - t_1) = \pi \). By using Equation (6.90) at \( t = t_{C-p} \):

\[
V_{n,C-p} = 2V_{ng}
\]

Design Control Curves

By substituting for the generalized parameters from Table 6.1 in the generalized equations, design equations for each topology in the family can be found. Using computer software, design curves can be derived. Fig. 6.41 shows the control characteristics curves for the ZCS-QRC Family. As an example, Fig. 6.42 shows the average and the \textit{rms} switch currents as a function of the voltage gain.

\textit{Fig. 6.41: DC voltage conversion ratio characteristics for:} (a) ZCS-QRC Buck, (b) ZCS-QRC Boost, (c) ZCS-QRC Buck-Boost, Cuk, Zeta, and Sepic

\textit{Fig. 6.42: Some of the ZCS-QRC Boots Main Switch (S) Normalized Stresses:} (a) Normalized Average Current, and (b) Normalized \textit{rms} Current.

6.6.4 The Basic Operation of the ZVS-QRC Cell

Figure 6.37(b) shows the generalized ZVS-QRC switching cell which is formed by adding the resonant capacitor \( C \) (which can be considered the switch internal capacitor), and the resonant inductor \( L \) to the conventional switching cell. As discussed before, this is also known as M-type resonant switch.
By applying this cell to the conventional DC/DC converters in Fig. 6.7, the ZVS-QRC family can be formed as shown in Fig. 6.43. In the next section, the modes of operation for the ZVS-QRC switching network will be discussed briefly and the main switching waveforms will be drawn.

Fig. 6.43: The DC/DC ZVS-QRC Family:
(a) Buck. (b) Boost. (c) Buck-Boost. (d) Cuk. (e) Zeta. (f) Sepic.

The typical switching waveforms for the ZVS-QRC cell are shown in Fig. 6.44. Table 6.3 shows the conduction status of the switches and diodes in each mode. As shown before, there are four modes of operation. It is assumed that before \( t = t_0 \), S was ON and D was OFF. The resonant inductor \( L \) was currying a current equal to \( I_F \) and the resonant capacitor \( C \) voltage was zero. **Mode 1** \(( t_0 \leq t \leq t_1 )\) starts when S is turned ON while D is OFF, which cause \( C \) to charge up linearly until its voltage reaches to a value equal to \( V_g \) at \( t = t_1 \) causing D to start conducting. The resonant inductor current during this mode doesn't change and it is equal to \( I_F \).

**Mode 2** \(( t_1 \leq t \leq t_2 )\) starts when D turns ON while S is OFF causing a resonant stage between \( C \) and \( L \) to start until the voltage of \( C \) tends to go negative forcing the switch diode \( D_s \) to turn ON at \( t = t_2 \) After this time, S can be turned ON at zero voltage.

![Main ZVS-QRC switching cell waveforms](image)

**Table 6.3: Switches conditions**

<table>
<thead>
<tr>
<th>Mode</th>
<th>( S )</th>
<th>( D )</th>
<th>( D_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>Mode 2</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
</tr>
<tr>
<td>Mode 3</td>
<td>ON</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>Mode 4</td>
<td>ON</td>
<td>OFF</td>
<td>Don’t care</td>
</tr>
</tbody>
</table>

**Mode 3** \(( t_2 \leq t \leq t_3 )\) starts when S is turned ON at zero voltage (Zero-Voltage Switching). The resonant inductor current starts charging up linearly until it reaches \( I_F \) causing D to turn OFF at zero current at \( t = t_3 \).

**Mode 4** \(( t_3 \leq t \leq t_0 + T_s )\) is a steady-state mode and nothing happen on it until S is turned OFF again to start the next switching cycle.

**Generalized Steady State Analysis**

From the description of the modes of operation, the equivalent circuit for each mode can be drawn as shown in Fig. 6.45. These equivalent circuits can be used along with the
description of the modes to write the mathematical Equations for each mode as follows (knowing that \( v_c(t_0) = 0 \) and \( i_i(t_0) = I_F \)):

**Mode 1: \([ t_o \leq t \leq t_1 ]\)**

\[
v_c(t) = \frac{I_F}{C}(t - t_0)
\]

\[
i_i(t_1) = I_F
\]

\[
v_c(t_1) = V_g
\]

\[
i_i(t_1) = I_F
\]

**Mode 2: \([ t_1 \leq t \leq t_2 ]\)**

\[
v_c(t) = V_g + Z_e I_F \sin \omega_o(t - t_1)
\]

\[
i_L(t) = I_F \cos \omega_o(t - t_1)
\]

\[
v_c(t_2) = 0
\]

\[
l_L(t_2) = I_F \cos \omega_o(t_2 - t_1)
\]

**Mode 3: \([ t_2 \leq t \leq t_3 ]\)**

\[
v_c(t) = 0
\]

\[
i_i(t) = I_F \cos \omega_o(t_2 - t_1) + \frac{V_c}{L}(t - t_2)
\]

\[
v_{cr}(t_3) = 0
\]

\[
i_L(t_3) = I_F
\]

**Mode 4: \([ t_3 \leq t \leq t_o + T_s ]\)**

\[
v_c(t) = 0
\]

\[
i_L(t) = I_F
\]

**Generalized Intervals Equations**

To simplify the analysis, the following time intervals are defined:

\[
\alpha = \omega_o(t_1 - t_0)
\]

\[
\beta = \omega_o(t_2 - t_1)
\]

\[
\gamma = \omega_o(t_3 - t_2)
\]

\[
\delta = \omega_o((t_0 + T_s) - t_3)
\]

These intervals can be derived as follows:

- From Equations (6.102) and (6.103):
\[ \alpha = \omega_o (t_1 - t_0) = \frac{V_g}{Z_o I_F} \]

By Using the normalized parameters defined earlier:
\[ \alpha = \omega_o (t_1 - t_0) = \frac{Q V_{ng}}{M I_{nF}} \]

- From Equations (6.104) and (6.105):
  \[ \beta = \omega_o (t_2 - t_1) = \sin^{-1} \left( -\frac{Q V_{ng}}{M I_{nF}} \right) \]
- From Equations (6.106) and (6.107):
  \[ \gamma = \omega_o (t_3 - t_2) = \frac{M I_{nF}}{Q V_{ng}} (1 - \cos \beta) \]
- From Fig. 6.44 and the above intervals:
  \[ \delta = \omega_o ((t_0 + T_s) - t_3) = \frac{2\pi}{f_{ns}} - \alpha - \beta - \gamma \]

**Fig. 6.45: The equivalent circuits for:**
(a) Mode 1, (b) Mode 2, (c) Mode 3, and (d) Mode 4

**Generalized Gain Equation**

The cell output to input generalized gain can be found using the average output diode $D$ voltage as follows:

\[
V_{D-ave} = V_{s-ave} - V_g \\
= \left[ \frac{1}{T_s} \int_{t_0}^{t_0 + T_s} V_s(t)dt \right] - V_g \\
= \frac{1}{T_s} \left[ \frac{I_F}{2C} (t_1 - t_0)^2 + V_g (t_2 - t_1) - \frac{Z_s I_F}{\omega_o} (\cos \omega_o (t_2 - t_1) - 1) \right] - V_g
\]

By using the normalized, we will have:

\[
V_{nD} = \frac{f_{ns}}{2\pi} \left[ \frac{M I_{nF}}{2Q} \alpha^2 + V_{ng} \beta + \frac{M I_{nF}}{Q} (1 - \cos \beta) \right] - V_{ng} \tag{6.108}
\]

By substituting for the generalized parameters ($V_{nD}$, $V_{ng}$, and $I_{nF}$) from Table 6.1 in Equation (6.108), we will have the gain equation for each converter in the family.
Generalized ZVS Condition

It can be noted from Fig. 6.43 that $S$ must be turned ON after $t = t_2$ and before $t = t_3$. In other words before $i_L(t) = I_F$ which cause $D$ to turn OFF $(i_D(t) = I_F - i_L(t))$ at $t = t_3$ causing $v_c(t)$ to start charging up linearly again. The generalized condition to achieve zero-voltage switching can be expresses as follows:

$$\alpha + \beta \leq \frac{2\pi}{f_{ns}} (1 - D) \leq \alpha + \beta + \gamma$$

Design Control Curves

By substituting for the generalized parameters from Table 6.1 in the generalized Equations, design Equations for each topology in the family can be found. Using computer software, design curves can be plotted. Fig. 6.46 shows the control characteristics curves for the ZVS-QRC family. Many other curves can also be plotted, as an example, Fig. 6.47 shows the average and the rms switch currents as a function of the voltage gain, $M$.

Fig. 6.46: DC voltage conversion ratio characteristics for: (a) ZVS-QRC Buck, (b) ZVS-QRC Boost, (c) ZVS-QRC Buck-Boost, Cuk, Zeta, and Sepic

Fig. 6.47: Some of the ZVS-QRC Buck-Boot Stresses: (a) Normalized Switch Peak Voltage, (b) Normalized Switch rms Voltage, (c) Normalized Switch Peak Current, and (d) Normalized Switch rms Current.

6.7 ZERO-VOLTAGE AND ZERO-CURRENT TRANSITION CONVERTERS

Previously converters operate with sinusoidal current through the power switches which results in high peak and rms currents for the power transistors and high voltage stresses on the rectifier diodes. Furthermore, when the line voltage or load current varies over a wide range, Quasi Resonant Converters are modulated with a wide switching frequency range, making the circuit design difficult to optimize. As a compromise between the PWM and resonant techniques, various soft-switching PWM converter techniques has been proposed recently to aim at combining desirable features of both the conventional PWM and Quasi Resonant techniques without a significant increase in circulating energy.

6.7.1 Switching Transition

To overcome the limitations of Quasi Resonant Converters, the Zero-Voltage-Transition (ZVT) or Zero-Current-Transition (ZCT) is the solution. Instead of using a series resonant network across the power switch, an alternative way is to use a shunt resonant network across the power switch. A partial resonance is created by the shunt resonant
network to achieve ZCS or ZVS during the switching transition. And it will still keep the advantages of a PWM converter because after the switching transition is over, the circuit reverts back to PWM operation mode.

The features of the ZCT-PWM and ZVT-PWM soft-switching converters are summarized in the following:

- Zero-current/voltage turn-OFF/ON for the power switch.
- Low voltage/current stresses of the power switch and rectifier diode.
- Minimal circulating energy.
- Constant-frequency operation.
- soft-switching for wide line and load range

One disadvantage is that the auxiliary switch does not operate with soft-switching, it is hard-switching, but the switching losses is much lower than a PWM converter. Another disadvantage is the transformer leakage inductance not utilized which is similar to Quasi-Resonant Converters. Therefore, the transformer should be designed with minimum leakage.

The ZVT and ZCT converters differ from a conventional PWM converters by the introduction of a resonant branch shown as Fig. 6.48. Figure 6.48(a) shows the ZVT-PWM switching cell, and in Fig. 6.48(b) shows the ZCT-PWM switching cell. $L$ is a resonant inductor, $C$ is a resonant capacitor, $S_1$ is an auxiliary switch, and $D_1$ is an auxiliary diode.

![Fig. 6.48(a) ZVT-PWM switching cell](image)

![Fig. 6.48(b) ZCT-PWM switching cell](image)

### 6.7.2 The Boost ZVT-PWM converters

In this section, we consider the boost-ZVT-PWM shown in Fig. 6.49 by replacing the ZVT-PWM switching cell shown in Fig. 6.48(a) into the boost converter which is shown in Fig. 6.48(b).

![Fig. 6.49(a) Boost-ZVT-PWM, (b) Simplified equivalent circuit](image)

The switching cycle is divide to six modes as shown in following.

**Mode I** $[t_o \leq t < t_1 ]$

![Fig. 6.50 Equivalent circuits for the seven modes of operation:](image)

(a) Mode I, (b) Mode II, (c) Mode III, (d) Mode IV,
(e) Mode V, (f) Mode VI and (g) Mode VII

![Fig. 6.51 Steady state waveforms For ZVT-Boost Converter](image)
Mode I as shown in Fig. 6.50(a) start at $t = t_0$ when auxiliary switch $S_1$ is turned ON. Since the main switch, $S$, and the auxiliary switch $S_1$ were OFF prior to $t = t_0$, it is clear that the diode, D must have been ON for $t < t_0$ to carry the output current. Hence, we assume the diode, D is ON and $D_1$ is OFF at $t = t_0$. So for $t > t_0$, $S_1$ is ON. The diode current starts to discharging and it reaches to zero when the inductor current $i_L$ is charging and reaches to the constant current source, $I_{in}$ at $t_1$. In this mode, the capacitor voltage, $v_c$ is equal to output voltage $V_o$ and also equal to the inductor voltage as given by,

$$V_o = L \frac{di_L}{dt} = v_c$$

From the above equation, the inductor current $i_L$ is given by,

$$i_L = \frac{V_o}{L} (t - t_0)$$

The above equation assumes zero initial condition for $i_L$.

As long as the inductor current is less than $I_{in}$, the diode will stay conducting and the capacitor voltage remains at $V_o$. At the time $t_1$, the inductor current becomes equal $I_{in}$, the diode, D stops conducting, and the circuit enters Mode II. From the above equation, we have,

$$I_{in} = \frac{V_o}{L} (t - t_0)$$

The time interval is given by

$$(t_1 - t_0) = \frac{L I_{in}}{V_o}$$

This is the inductor current charging state.

**Mode II** [$t_1 \leq t < t_2$]

Mode II starts at $t_1$ when the diode, D is OFF, circuit as shown in Fig. 6.50(b) resulting in a resonant stage between $L$ and $C$. During the time between $t_1$ and $t_2$, the main switch, $S$ remains OFF, and the $S_1$ is still ON, but the both diodes are OFF. The initial capacitor voltage is still $V_o$, but the initial $i_L$ has changed to $I_{in}$. The first order differential equation that represent this mode are given by,

$$C \frac{dv_c}{dt} + I_{in} = i_L$$

$$v_c = L \frac{di_L}{dt}$$

$$\frac{d^2i_L}{dt^2} - \frac{1}{LC} i_L = \frac{1}{LC} I_{in}$$

$$\text{(6.109)}$$
The solution for \( i_L \) and \( v_c \) is given by,

\[
i_L = \frac{V_o}{Z} \sin \omega_o (t - t_1) + I_{in}
\]

\[
v_c = V_o (2 - \cos \omega_o (t - t_1))
\]

The time interval between \( t_1 \) and \( t_2 \) is given by

\[
(t_2 - t_1) = \frac{1}{\omega_o} \cos^{-1} (2)
\]

The diode voltage starts to charging up due to the decreasing voltage.

\[
v_d = V_o - v_c
\]

Substitute the \( v_c \), diode voltage becomes,

\[
v_d(t) = -V_o + V_o \cos \omega_o (t - t_1)
\]

**Mode III \([t_2 \leq t < t_3]\):**

Mode III starts when the capacitor discharging to zero. At this modem the main switch, \( S \) remains \( OFF \), the auxiliary switch, \( S_1 \) is still \( ON \), and both diodes are \( OFF \).

Now,

\[
v_c(t) = 0
\]

**Mode IV \([t_3 \leq t < t_4]\):**

Mode IV starts at \( t = t_3 \), when the main switch, \( S \) is turned \( ON \) and the auxiliary switch, \( S_1 \) is turned \( OFF \). At \( t_3 \), the initial capacitor voltage is zero, and the inductor starts linearly discharging from \( i_L(t_2) \) to zero during \( t_3 \) to \( t_4 \). The diode, \( D \) remain \( OFF \) since its voltage is negative, but the diode, \( D_1 \) turns \( ON \) at \( t = t_3 \) to carry the inductor current.

The input voltage is equal to inductor voltage, and the output voltage is equal to negative inductor voltage, \( v_i \).

\[
v_{in} = v_f = L_f \frac{di_f}{dt}
\]

\[
v_o = -v_i = -L_i \frac{di_i}{dt}
\]

\[
i_L(+) = V_o (t - t_2) + i_L(t_2)
\]

The current \( i_{Lo} \) which is current \( I_{in} \) can be expressed as,
\[ I_{in} = \frac{1}{L_o} V_{in} (t_4 - t_2) + I_{in} \]

Now the final value of \( i_i(t) \) is zero and the initial value of the capacitor, \( v_c(t) \) is zero. The input current goes through the main switch S.

**Mode V \([t_4 \leq t < t_5]\):**

At this mode both switches are OFF, and the diodes starts are OFF also at \( t = t_4 \). The inductor current is zero, and the input current is only going thru the capacitor,

\[ I_{in} = C \frac{dv_c}{dt} \]

The capacitor voltage can be expressed as,

\[ v_c(t) = \frac{1}{C} I_{in} (t_5 - t_4) \]

The capacitor is charging up from zero and will be reaches to output voltage at \( t = t_5 \). The time interval is,

\[ (t_5 - t_4) = \frac{V_o C}{I_{in}} \]

then it enters the mode VI at this point.

**Mode VI \([t_5 \leq t < t_6]\):**

When the capacitor reaches to the output voltage, the diode, D starts conduction, but At this mode, both of the switches are still OFF. The diode current will reach the input current immediately. At \( t = t_5 \), the capacitor is equal to the output voltage until the auxiliary switch is turned ON again, then the cycle will repeated from mode I. The waveforms for the six modes of operation are shown in Fig.6.51.

**PROBLEMS**

**ZCS Quasi Resonant Buck Converters:**

6.1 Consider a ZCS buck converter whose steady-state waveforms are shown in Fig. 6.10 Assume that the converter has the following parameters: \( V_{in}=25V \), \( I_o=1A \), \( L=3 \ \mu H \), \( C=0.02\mu F \). Determine the time intervals at \( t_1, t_2, t_3 \).

6.2 Consider the ZCS buck converter shown in Fig. P6.2. Determine the output power.
6.3 Determine M and Q for a ZCS buck converter with L-type switch that has the following converter components: \( L = 15 \, \mu H, \, C = 60 \, \mu F, \, f_s = 100kHz, \, V_{in} = 40V \) and \( I_o = 0.74A. \)

6.4 Consider the ZCS buck converter shown in Fig. P6.4(a) with \( L=20\mu H, \, C=5\mu F, \, V_{in} = 50V, \, V_o = 30V, \, I_o = 20A. \) Assume that \( \frac{L_o}{R_o} \gg \frac{1}{T} \) and the switching waveform of the transistor is shown in Fig. P6.4(b). Determine the minimum \( t_{ON} \) in \( \mu s \) to achieve ZCS and determine the switching period \( T. \)

6.5 Repeat Problem P6.4 by adding a diode across the transistor to allow bi-directional current flow to produce full-wave ZCS buck converter. Assume all values are the same as in Problem P6.4.

D6.6 Consider a ZCS buck converter with L-type switch with \( M=0.35 \) and \( Q=1. \) Design for the resonant tank \( L \) and \( C \) for \( V_{in} = 40V, \, f_s = 250 \, kHz \) and \( I_o = 0.7A. \)

D6.7 Design for \( L \) and \( C \) and draw the waveforms for the ZCS buck converter shown in Fig P6.7 that has the following design parameters: \( V_{in}=25V, \, V_o=12V, \, f_s= 250kHz, \, I_o=1A, \) and \( f_o=625kHz. \) Draw the steady-state waveforms for \( i_L, \, i_C, \, v_{SW}, \, v_D, \, i_D, \, i_o. \)

D6.8 Consider a ZCS buck converter with a uni-directional switch and the following specifications:
  - Maximum load power \( 750W. \)
  - Nominal output voltage \( 5V. \)
  - Nominal input voltage \( 12V. \)
  - Switching frequency \( 85kHz. \)
  - Maximum peak resonant inductor current twice the average load current.

  (a) Design for the resonant tank \( L \) and \( C. \)
  (b) If the load current change by \( \pm 50\% \) what is the new minimum and maximum switching frequencies required to maintain the output voltage at \( 5V. \)
  (c) Repeat part (b) for the variation of \( \pm 20\% \) in the average input voltage.

D6.9 Design a ZCS buck converter of the L-type switch type that has the following parameters: \( V_{in} = 50V, \, f_s = 100kHz, \, I_o = 0.2A, \, V_o = 49V, \) and \( f_{ns} = 0.5. \) Design for \( L, \, C, \, L_o, \, C_o, \) and \( R. \) Design for an output ripple current within \( 15\% \) of its dc value and an output ripple voltage not to exceed \( 2\%. \)

6.10 Consider the ZCS buck converter shown in Fig. P6.10 with the following parameters: \( L = 25V, \, C = 4.7\mu F, \, V_{in} = 58V, \) and \( I_o = 18A. \) Determine the output voltage and \( f_s \) to deliver an average output power equals to \( 580 \) Watts.
**ZCS Quasi Resonant Boost Converters:**

D6.11 Design a ZCS boost converter for the following parameters: $V_{in} = 20V$, $V_o = 36V$, $I_o = 0.7A$ and $f_s = 250kHz$. What is the range of $f_s$ needed to keep $V_o$ constant when $I_o$ changes between 0.2A to 1A.

D6.12 Design a ZCS boost converter with M-type switch with following design parameter: $M=2.73$ and $f_{ns}=0.6$. It is desired to deliver 0.5A load current to $V_o=68V$.

D6.13 Design an M-type switch boost Quasi-resonant converter with the following parameters: $M = 1.8$, $Q = 2.5$, $f_{ns} = 0.5$. The input voltage varies between 18V to 26V, and $I_o$ varies between 0.5A to 1A. What is the range of $f_s$ to keep the converter output voltage constant at $V_o = 38V$.

**ZCS Quasi Resonant Buck-boost Converters:**

6.14 Consider a ZCS buck-boost converter with L-type converter shown in Fig. P6.14. Determine $M$, $Q$ and $f_s$. What is the new $f_s$ needed to keep $V_o$ constant if the average load changes to 1.8A.

6.15 Determine the value of $I_f$ in the ZCS buck-boost converter given in Fig. P6.15.

6.16 Consider a ZCS buck-boost with L-type switch to be designed for the following specifications: $V_{in} = 40V$, $V_o = 20V$, $I_o = 4A$ and $f_s = 250kHz$.

6.17 Design a ZCS buck-boost converter with L-type converter for the following specifications: $V_{in} = 30V$, $V_o = 20V$, $I_o = 2.8A$ and $f_s = 50KHZ$.

6.18 Design a ZCS buck-boost converter with L-type switch to operate at 150kHz and delivers 48W to $V_o=47V$ from a 10V dc source.

6.19 Figure P6.19 shows a ZCS buck-boost with S is being unidirectional. (a) Sketch $i_L$ and $v_c$, (b) Discuss the four modes of operation, and (c) Derive the expression for $i_L$ and $v_c$ in terms of circuit parameters.

6.20 Repeat Problem P6.19 by using a ZCS buck-boost L-type converter. Select any set of $M$, $Q$, $f_{ns}$ you see fit for your design.

**ZVS Quasi Resonant Buck Converters:**

6.21 Derive the voltage gain expression for the M-type ZVS buck converter.

6.22 Consider the buck ZVS shown in Fig. P6.22 with an alternative way of implementing the L-type resonant switch. Assume $L_o/L$ is very large.

(a) Discuss the modes of operation over one-switching cycle.
(b) Draw typical waveform for $i_L$ and $v_c$.
(c) Derive the expression for $\frac{V_o}{V_{in}}$.

6.23 Consider the resonant capacitor voltage and inductor current shown in Fig. P6.23 for a ZVS buck. Determine L, C, $t_1$, $t_3$, $V_o$, $Q$, $f_{ns}$, M, and $i_L(t_2)$.

**ZVS Quasi Resonant Boost Converters:**

6.24 Consider the ZVS boost Quasi-Resonant converter shown in Fig. P6.24. Assume $S$ is bi-directional.
(a) Sketch the waveforms for $v_c$ and $i_L$.
(b) Discuss the four modes of operation.
(c) Derive the following expression for the voltage gain:

$$M = \frac{V_o}{V_{in}} = \frac{1}{f_{ns} \frac{f_s}{f_o} \left(\alpha - \frac{Q}{2M} + \frac{M}{Q} (1 - \cos \alpha)\right)}$$

where,

$$\sin \alpha = -\frac{Q}{M}, \quad f_{ns} = \frac{f_s}{f_o}, \quad Q = \frac{R_o}{Z_o},$$

$$Z_o = \sqrt{\frac{L}{C}}, \quad f_o = \frac{1}{2\pi \sqrt{LC}}$$

D6.25 Design a ZVS boost converter with the following specifications:
- Maximum load power 750W
- Nominal input voltage 12V.
- Nominal input voltage 5V.
- Switching frequency 100kHz.
- Maximum peak capacitor voltage is 1.5 times average input voltage.

6.26 Figure P6.26(a) shows isolated boost converter with switches that generate at 50% duty cycle. If it is assumed that $L_{in}$ and $C_o$ are large, then the input current, $I_{in}$, and, $V_o$, may be assumed constants as shown in Fig. P6.26 (b). The switching waveforms for S1 and S2 are shown in Fig. P6.26(c). It can be shown that there are four modes of operation in steady state with $S_2$ operates with ZVS.
(a) Discuss the modes of operation.
(b) Show that $\frac{I_o}{I_{in}} = n(1 - D)$ where $I_{in}$ and $I_o$ are the average input and output currents.

**ZVS Quasi Resonant Buck-Boost Converters:**
Determine the output voltage for the ZVS converter shown in Fig. P6.27 and sketch the waveforms for $i_L$, $v_c$, $i_D$.

Design a buck-boost ZVS converter with the following steady state operating point $M = 0.55$, $f_{ns} = 0.3$, $Q = 0.1$. Assume $V_{in} = 30V$ and $T_s = 10\mu s$ and the output current is at least 1A.

**General Soft-switching Converters:**

Figure P6.29 shows a Zero-Current-Transition (ZCT) buck converter.
(a) Discuss the modes of operation and show the typical waveforms for all currents and voltages. Assume $I_o$ is constant.
(b) Give the expression for the circulating energy in the resonant tank.
(c) What are the major features of the converter?

Consider the circuit given in Fig. P6.30 that operates in ZVS mode with S1 and S2 operate alternatively. Discuss the operation of the circuit and sketch the waveforms for $i_{sw}$, $i_L$, $v_L$, $i_{im}$, $i_o$ and $v_s$.

Figure P6.31 (a) shows a ZVS soft switching capacitor voltage clamped converter. Assume $C_o >> C_1$ and $C_o >> C_2$. It can be shown that in steady-state there are four modes of operation. Assume $S_1$ and $S_2$ are switched as shown in Fig. 6.31(b).
(a) Sketch the waveforms for $i_L$, $v_{c1}$, $v_{c2}$ and $i_{D2}$.
(b) Discuss the four modes of operation.
(c) Derive the output voltage gain in terms of the circuit parameters.

Figure P6.32 shows a soft switching technique that is based on the concept of clamped current (CC) soft switching. Such converter family is known as a ZCS Quasi-square wave resonant converter. It is assumed that $L_{eq}$ is large enough so its current may be represented as a current source. $S$ and $D_s$ form unidirectional switch. This topology offers several advantages. Discuss these advantages.
(a) Sketch the steady state waveforms for $i_L$, $i_{sw}$, $i_D$, $v_c$, $v_o$ . Assume $I_f < I_o$ and the first mode starts at $t = t_0$ when $S$ is ON and $D$ is ON initially. It can be shown that there are 4 modes of operation.
(b) Show that during the resonant mode (Mode 3), the resonant inductor current and resonant capacitor voltage equations are given by,

\[
\begin{align*}
\dot{i}_L(t) &= I_f \left( \cos \omega_o (t - t_2) + 1 \right) \\
v_c(t) &= V_{in} + Z_o I_f \sin \omega_o (t - t_2)
\end{align*}
\]

where $Z_o = \sqrt{L/C}$ and $t = t_2$ is the starting of Mode 3.

Figure P6.33 shows a soft switching converter cell with the diode replaced by a switch $S_D$. By allowing the rectifier to become a controllable switch, the converter voltage gain can be controlled by the pulse width of $S$. This soft-switching family is
known as PWM quasi-square wave converters (PWM-QSW). Unlike the quasi-
resonant converter, which is frequency controlled converter, the QSW converters
are PWM control at constant frequency. Discuss the four modes of operation.

6.34 Determine the average output voltage for the ZCS buck converter shown in Fig.
P6.34.